Evaluation of Correlations for Natural Gas Compressibility Factors

by

Mohammed Najim Al-Khamis

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

PETROLEUM ENGINEERING

March, 1995

INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313/761-4700 800/521-0600

Evaluation of Correlations for Natural Gas Compressibility Factors

BY

MOHAMMED NAJIM AL-KHAMIS

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

In

PETROLEUM ENGINEERING

March 1995

UMI Number: 1374693

UMI Microform 1374693
Copyright 1995, by UMI Company. All rights reserved.

This microform edition is protected against unauthorized
copying under Title 17, United States Code.

UMI
300 North Zeeb Road
Ann Arbor, MI 48103

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Dhahran. Saudi Arabia

College of Graduate Studies

This thesis, written by Mohammed Najim Al-Khamis under the direction of his Thesis Advisor and approved by his Thesis Commitee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in petroleum engineering.

Thesis Committee

M. A. Marhoun

Dr. Muhammed Al-Marhoun
Thesis Advisor

H. Y. Al-Yousef

Dr. Hassan Y. Al-Yousef
Member

Dr. Habib. Menouar
Member

Dr. Khalid Al-Fossail
Department Chairman

Dr. Ala H. Al-Rabeh
Dean, College of Graduate Studies

Date

DEDICATION

To my loving Family
and my Parents

ACKNOWLEDGMENT

Praise and gratitude be to the Almighty, the creator and sustainer of the Universe, and peace be upon Prophet Muhammad, his progeny and faithful companions.

My great acknowledgment to King Fahd University of Petroleum and Minerals for the support in this research. In addition, I would like to express my appreciation to Professor Muhammad Al-Marhoun as my major advisor and for his tremendous support and guidance. I also wish to thank the other members of my Thesis Committee Dr. H. Al-Yousef and Dr. H. Menouar. Finally, gratefulness to H. Al-Awami, I. Iliyan and A. Al-Mubarak of Saudi Aramco for their continuous assistance and encouragement.

TABLE OF CONTENTS

Page #

List of Tables

List of Figures

ɔɩ F U

List of Figures (Cont.)

جامعة الملك فهد للبترول والتعدين
الظهران - المملكة العربية السعودية

دراسة مقارنة للعلاقات الرياضية لمعامل التضاغط للغاز الطبيعي

درجة الماجستير في العلوم

محمد نجم آل خميس

خلاصة الرسالة

لأكثر من أربعة عقود ، عديدة من العلاقات الرياضية قد استخدمت في الحسابات البترولية لتقدير معامل التضاغط للغاز الطبيعي (معامل Z) وهذه الرسالة تقارن تسع من العلاقات الرياضية الأكثر استخداماً مستعملاً التحليل الخطئي والتحليل بالرسم البياني . وقد استخدم في المقارنة اجمالياً ٥٩٤٠ نقطة بيانية وقبل تقييم العلاقات الرياضية تم اختبار نقاط البيانات لاحتمالات التباين أو التناقض مع الرسوم البيانية الأخرى ، وتم ابراز النتائج . وبناء علي ذلك فإن بيانات أدنى منحنيين قد تم تنقيحهما .

من خلال التسع علاقات التي تم دراستها فإن علاقة درانشك وأبو القاسم وجد أنها تعطي أقل متوسط خطأ نسبي مطلق بالنسبة للنقاط البيانية الأصلية . في المقابل فإن علاقة هول وياربوخ قد أعطت أقل متوسط خطأ نسبي مطلق بالنسبة للبيانات المنقحة ، وبالإضافة فإن العلاقتين وجد أنهما يبينان إلي حد كبير الشكل الفعلي للعلاقة .

مارس ١٩٩٥

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
Dhahran, Saudi Arabia

Evaluation of Correlations for Natural Gas Compressibility Factors

MASTER OF SCIENCE DEGREE
By
Mohammed Najim Al-Kahmis

THESIS ABSTRACT

For more than four decades, several correlations have been used in petroleum calculations to estimate the Natural Gas compressibility factor (Z-factor). This paper compares nine of the most commonly used correlations using error and graphical analysis. A total of 5940 data points were used in the comparison. Prior to correlations evaluation the data were tested for possible discrepancy or inconsistency with other charts, and the results were highlighted. Consequently the data of the lowest two curves T_r= 1.05 and 1.10 were smoothed.

Among the nine correlations studied, Dranchuk and Abou-Kassem correlation was found to give the least average absolute relative error with respect to original data. On the other hand, Hall and Yarborough correlation gave the least average absolute relative error with respect to smoothed data. In addition, both correlations were found to describe the shape of the actual function very closely.

March 1995

CHAPTER 1

Introduction

The Natural Gas compressibility factor (Z factor) is often required in many petroleum calculations. The importance of this gas property in evaluating gas reserves, gas flow in pipes, gas reservoirs simulation, material balance calculations and gas well tests can not be over emphasized. Z factor values are normally obtained from charts or tables of experimental data or estimated by empirical correlations.

For several years the Standing-Katz Z-factor chart or its tabulated[1] form have been used as a reliable source for Z-factor estimation for mixtures composed mainly of pure hydrocarbon gases. The use of this chart in complex calculations, however, is often time consuming. For this reason, several correlations have been developed to represent part of or the whole chart. The correlations can be classified as direct or iterative. The accuracy, rate of convergence, and computer memory requirements of each correlation differ from one to the other. In addition, the range of applicability could be different.

In 1975, Dranchuk and Abou-Kassem[2] developed an iterative correlation to calculate Z factor. They also compared their correlation with two other iterative correlations. In the comparison, they used 2940 data points obtained from the Standing-Katz table as well as some other sources. The authors compared the three correlations using the overall average absolute relative error of each correlation.

In 1989, Takacs[3] compared thirteen different correlations used to calculate Z factors. He used 180 Z factor values obtained from the Standing-Katz chart in the comparison. The data were in the ranges of $1.2 \leq T_r \leq 3.0$ and $0.2 \leq P_r \leq 15.0$. He calculated the average absolute relative error with respect to 10 pseudo reduced temperatures and 18 pseudo reduced pressures. In addition, he calculated the over all average absolute relative error and the average running time of each method.

Thesis Objectives

This thesis evaluates and compares nine of the most commonly used correlations by two means: Error analysis and Graphical analysis. 5940 Z factor values obtained from the Standing-Katz table were used in the analysis. The Z factor data are in the ranges of $1.05 \leq T_r \leq 3.0$ and $0.2 \leq P_r \leq 15.0$. Prior to calculating the errors the data were validated by plotting and typographical error points were corrected. In addition, the data were verified to make sure it is correct, smooth and consistent with the approved theories. The average absolute relative errors with respect to 20 pseudo reduced temperatures and with respect to 297 pseudo reduced pressures were then calculated. In addition, some statistical data were calculated for each correlation and for different ranges of T_r and P_r.

For a better understanding of each correlation representation, 3-D plots of each correlation were constructed. The plots were then compared graphically with a similar plot of the actual function.

History of The Compressibility-Factor Chart

Work on the PVT behavior of gases started as early as 1870 by Andrew and Amagat[4]. The work was then continued by several investigators till 1940, when Brown and Holcomb[5] first presented the Natural Gas compressibility-factor chart. Their chart was extended only to a reduced pressure of 8 due to scanty of the data.

The gas compressibility-factor was studied later by Standing and Katz[1] using 16 saturated vapors in equilibrium with a crude oil. A total of sixteen data points were obtained. Standing and Katz reported the Natural Gas gravity as 0.596 and the gas consisted of 93% Methane, 4.25% Ethane, 1.61% Propane, 0.43% Nitrogen and 0.51% Carbon dioxide. The experimental temperature and pressure ranges were 35 to 250 °F and 1000 to 8220 psia respectively.

Standing and Katz compared their data with the chart of Brown and Holcomb and found both to be in excellent agreement up to reduced pressures of about 5. The data were found to deviate from the chart at higher pressures, and were found to be closer to the values from the chart based on pure Methane gas. Since the original chart did not cover the pressure range of their study and it was found to deviate consistently above a reduced pressure of 5, a revised and extended plot was prepared. The new chart is identical with the original chart up to reduced pressures of 4. The new chart was constructed based on the sixteen experimental data points obtained and by using the Methane gas chart as a guide. In summary, no mixtures having molecular weights in excess of 40 were included in preparing this chart although mixtures of cyclohexane and benzene as well as the paraffin series and up to three or four per cent of nitrogen were included[6]. The new chart was drawn up to a reduced pressure of 15 (Figure 1).

Comments on Natural Gas Z-Factor Chart:

Since the samples used by Standing and Katz are in equilibrium with crude oil, the composition of the free gas will then vary depending on the applied condition of pressure and temperature. This could be also explained by the variations in the calculated gas specific gravities of the experimental data points of Standing and Katz. In addition, the data at high pressures were found to be closer to the chart for pure Methane gas basically because at high pressures mostly Methane gas will be in gaseous condition.

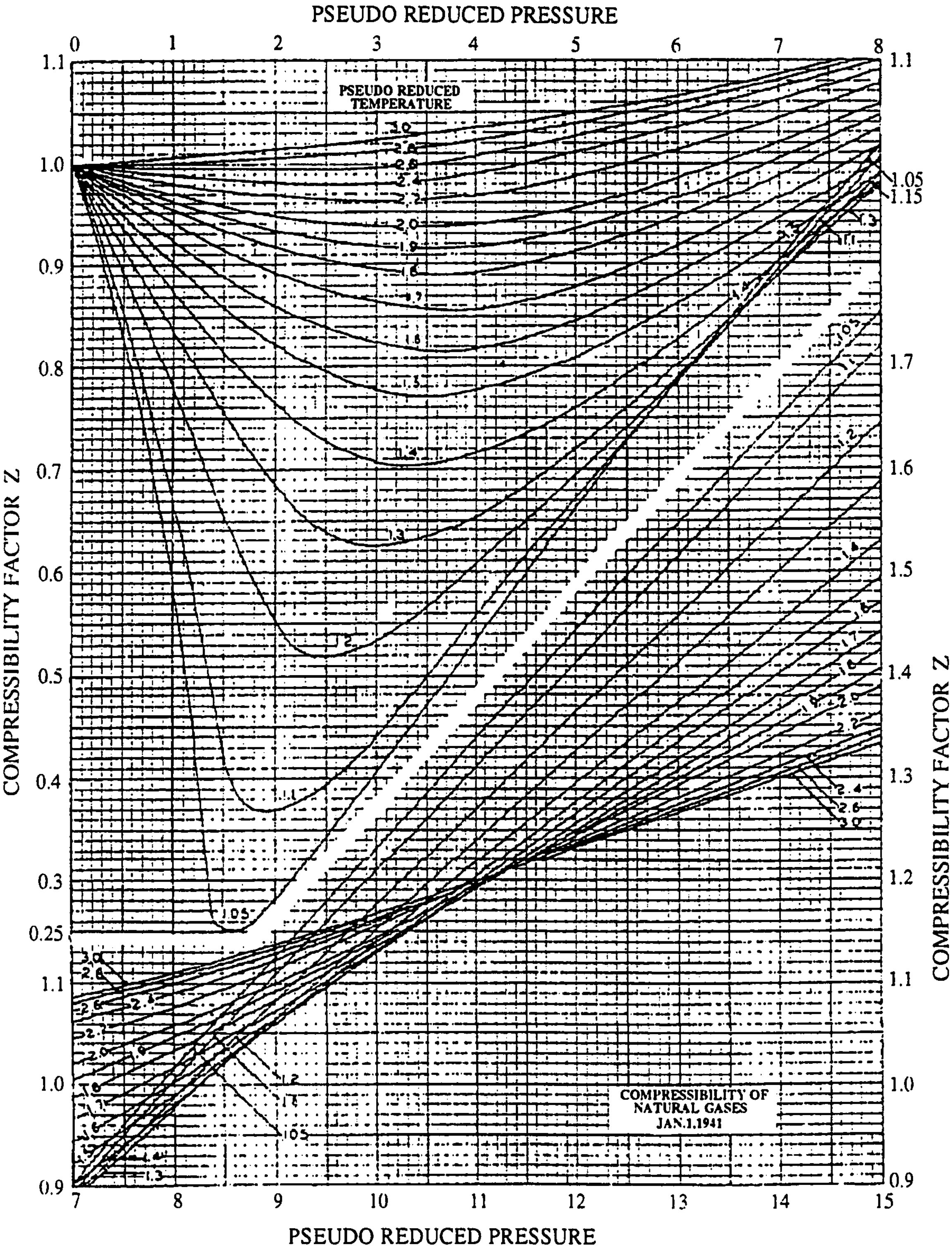

Figure 1. Standing and Katz Natural Gas Z factor chart.

CHAPTER 2

Gas Compressibility-Factor Correlations

For years, engineers have used empirical correlations in lieu of the existing tables and charts for determining the gas compressibility factor. The effective use of the correlations, however, lies in an understanding of the way they were derived and a knowledge of their limitations. This section presents a review of the nine correlations investigated. The correlations are divided into two groups: direct and iterative relations.

Direct Relations

Leung (1964)[7]. The Standing-Katz Z-factor chart was fitted by the following mixed power polynomial and the coefficients of correlation were determined by the method of least squares fitting:

$$Z = \sum_{i=1}^{4}\sum_{j=1}^{4} B_{ij}\ P_r^{(i-1)}\ T_r^{(1-j)} \tag{2.1}$$

Where B_{ij} is 4 X 4 matrix equal to:

$$\begin{vmatrix} 1.877 & -4.936 & 8.987 & -5.215 \\ -0.6562 & 3.692 & -6.477 & 3.077 \\ 0.1015 & -0.5242 & 0.8359 & -0.3192 \\ -0.00422 & 0.0205 & -0.0288 & 0.00742 \end{vmatrix}$$

Leung stated that his correlation permits the estimation of Z within approximately 1% in the following ranges:

$$0.5 \leq P_r \leq 11.0$$

$$1.10 \leq T_r \leq 2.6$$

Papay (1968)[8]. Papay proposed the following equation to calculate Z-factor:

$$Z = 1 - (P_r / T_r) [0.36748758 - 0.04188423 P_r] \quad (2.2)$$

As can be realized that the Z value is equal to one when P_r is equal to zero which is consistent with the chart. In addition, Papay's equation is simple and has two constants only.

Gopal (1977)[9]. The author initially divided Standing-Katz Z-factor chart into two parts by drawing a line isobarically at a P_r value of 5.4. The region between P_r value of 0.2 and 5.4 was further divided into twelve ranges of pseudo reduced pressures and temperatures. Each range was then curve-fitted to the following general equation:

$$Z = a_1 P_r + a_2 P_r T_r + a_3 T_r + a_4 \quad (2.3)$$

Where a_1 to a_4 are constants. A total of forty eight constants were obtained for the twelve ranges as follows:

P_r Range	T_r Range	a_1	a_2	a_3	a_4
$0.2 \leq P_r < 1.2$	$1.05 \leq T_r < 1.2$	1.6643	-2.2114	-0.3647	1.4385
	$1.2 \leq T_r < 1.4$	0.5222	-0.8511	-0.0364	1.0490
	$1.4 \leq T_r < 2.0$	0.1391	-0.2988	0.0007	0.9969
	$2.0 \leq T_r < 3.0$	0.0295	-0.0825	0.0009	0.9967
$1.2 \leq P_r < 2.8$	$1.05 \leq T_r < 1.2$	-1.3570	1.4942	4.6315	-4.7009
	$1.2 \leq T_r < 1.4$	0.1717	-0.3232	0.5869	0.1229
	$1.4 \leq T_r < 2.0$	0.0984	-0.2053	0.0621	0.8580
	$2.0 \leq T_r < 3.0$	0.0211	-0.0527	0.0127	0.9549
$2.8 \leq P_r < 5.4$	$1.05 \leq T_r < 1.2$	-0.3278	0.4752	1.8223	-1.9036
	$1.2 \leq T_r < 1.4$	-0.2521	0.3871	1.6087	-1.6635
	$1.4 \leq T_r < 2.0$	-0.0284	0.0625	0.4714	-0.0011
	$2.0 \leq T_r < 3.0$	0.0041	0.0039	0.0607	0.7927

For P_r values greater than 5.4 and the whole range of T_r, data was best described by the following equation:

$$Z = P_r\,(0.711 + 3.66\,T_r)^{-1.4867} - 1.637 / (0.319\,T_r + 0.522) + 2.071 \qquad (2.4)$$

In conclusion, Gopal stated that the thirteen equations do not only duplicate the whole Standing and Katz Natural Gas Z-factor chart, but also predict Z-factor values beyond the range of the chart as compared with results obtained from the Hall and Yarborough method.

Burnett (1979)[10]. The following formula was developed to approximate the gas compressibility factor given by the American Gas Association (AGA):

$$Z = 1 + (Z' - 1) (\sin 90\, U)^N \tag{2.5}$$

Where Z', U and N are equal to:

$$Z' = 0.3379 \ln (\ln T_r) + 1.091$$

$$P_r' = 21.46\, Z' - 11.9\, Z'^2 - 5.9$$

$$U = P_r / P_r'$$

$$N = [\, 1.1 + 0.26\, T_r + (1.04 - 1.42\, T_r)\, U \,]\, (\exp(U) / T_r)$$

While testing, Burnett observed differences between Standing and Katz Natural Gas Z factor data and AGA data. Standing and Katz Z factor data were found to have lower values than AGA data. Upon observing this, Burnett decided to revise his formula above to track AGA data.

Burnett stated that the accuracy of the equation diminishes for pseudo reduced temperatures below 1.3 and above 1.85 and for pseudo reduced pressures such that $P_r > P_r'$. His correlation also gives a Z value of one when P_r is equal to zero which is consistent with the data.

Papp (1979)[11]. The author proposed the following equation to calculate the compressibility factor:

$$Z = 1 + R_1\, P_r + R_2\, P_r^2 - W R_3\, P_r / (P_r^2 + R_6\, P_r + R_7) \tag{2.6}$$

Where R_i is a function of pseudoreduced temperature only and thus needs to be calculated only once for each pseudoreduced temperature. The variable W is a function of both pseudoreduced temperature and pressure. The values of R_i and W are as follows:

$$R_1 = -3.23716 \text{ X } 10^{-3}\, T_r^5 + 4.0734 \text{ X } 10^{-2}\, T_r^4 - 0.202996\, T_r^3 + 0.494968\, T_r^2 - 0.573187\, T_r + 0.244275$$

$$R_2 = (17.5648\, T_r^3 + 185.316\, T_r^2 - 337.652\, T_r + 420.016)^{-1}$$

$$R_3 = T_r^3 / (86.1372 - 235.563\, T_r + 256.036\, T_r^2 - 135.986\, T_r^3 + 34.2576\, T_r^4 - 2.7089\, T_r^5)$$

$$R_4 = 9.6283\, T_r^2 - 19.7803\, T_r + 11.9919$$

$$R_5 = (6.83622 - 22.0656\, T_r + 14.3817\, T_r^2 + 2.081973\, T_r^3) / T_r^3$$

$$R_6 = R_3 R_4 - 2 R_5$$

$$R_7 = R_5^2$$

When P_r is less than or equal to R_5, W is equal to one. If P_r is greater than R_5 then:

$$W = 1 - R_{10} (P_r - R_5)^2 (P_r - R_8)(P_r - R_9)^2$$

$$R_8 = R_5 + 3.7 \text{ X } 10^{-10} (15.1 - R_5) / (T_r - 1.1)^9$$

$$R_9 = 15.1 + 2 (15.1 - R_5)(15.1 - R_8) / (45.3 - 2 R_8 - R_5)$$

$$R_{10} = [(15.1 - R_5)^2 (15.1 - R_8)(15.1 - R_9)^2]^{-1}$$

Papp here also forces the correlation to give a Z value of one when P_r is equal to zero. Moreover, he stated that the correlation is valid for the following ranges:

$$0.2 \leq P_r \leq 15.0$$

$$1.2 \leq T_r \leq 3.0$$

Iterative Relations

Hankinson, et al. (1969)[12]. The authors correlated the Natural Gas compressibility factor using the Benedict-Webb-Rubin equation of state. The data was split into two regions in order to improve the accuracy of the data representation; one for pseudo reduced pressures less than 5.0 and the other for pseudo reduced pressures between 5.0 and 15.0. The first region coefficients were fitted based on 252 data points while the other were based on 328 data points. Thus, two sets of coefficients were obtained one for each region. The proposed equation has the following form:

$$1/Z - 1 + [a_1 T_r - a_2 - a_3 / T_r^2] (P_r/(Z^2 T_r^2)) + (a_4 T_r - a_5) (P_r^2/(Z^3 T_r^3)) + a_5 a_6 a_7 P_r^5/(Z^6 T_r^6) [1 + a_8 P_r^2/(Z^2 T_r^2)] \exp[- a_8 P_r^2/(Z^2 T_r^2)] = 0 \quad (2.7)$$

The constants a_1 to a_8 of each region are as follows:

Coefficient	$0.4 \leq P_r < 5.0$	$5.0 \leq P_r \leq 15.0$
a_1	0.12215481	0.11812287
a_2	0.38193005	0.37922269
a_3	0.027271364	0.19845016
a_4	0.022199287	0.024181399
a_5	0.001290236	0.0014507882
a_6	-0.015674794	0.037905663
a_7	0.023834219	0.048911693
a_8	0.43617780	0.0631425417

Equation (7) can be solved for the compressibility factor using iteration techniques. Hankinson, et al. stated that the correlation was able to reproduce the tabular compressibility factor data of Poettman and Carpenter(13) with an average error of less than 2 % for pseudo reduced pressures less than 5.0, and 4 % for pseudo reduced pressures between 5.0 and 15.0. The maximum calculated errors were found to occur in the second region and at low T_r values. Hankinson, et al. suggested that the proposed correlation be used only at T_r's above 1.1. In conclusion, the correlation is suggested to be used in the following ranges:

$$0.4 \leq P_r \leq 15.0$$
$$1.1 \leq T_r \leq 3.0$$

<u>Comments on the Correlation:</u>

Hankinson, et al. correlation was tested for points within the Standing and Katz chart range and outside the valid range of the correlation. It was found that Newton Raphson iteration technique failed to converge for seven points T_r= 1.05 and P_r= 0.70, 0.75, 0.80, 0.85, 0.90, 0.95 and 1.00. In this case bisection method was used to solve the correlation for the value of Z factor. In addition, the correlation was found not to possess any Z factor root within the limits of the chart for eight points T_r= 1.05 and P_r= 0.20, 1.25, 1.30, 1.35, 1.40, 1.45, 1.50 and 1.55.

Hall and Yarborough (1973)(14,15). Hall and Yarborough have fitted data taken from the Standing and Katz Z-factor chart into an expression based on the

Starling-Carnahan equation of state. The proposed mathematical form is as follows:

$$Z = 0.6125\,(P_r/(y\,T_r))\,\exp[-1.2\,(1 - 1/T_r)^2] \qquad (2.8)$$

Where:

$$y = b\,\rho\,/\,4$$

Where b is van der Waals covolume and ρ is density. The value of y can be obtained as the solution of the following non-linear relation:

$$\begin{aligned} f(y) = & -0.06125\,(P_r/T_r)\exp[-1.2(1-1/T_r)^2] + (y+y^2+y^3+y^4)/(1-y)^3 \\ & -(14.76/T_r-9.76/T_r^2+4.58/T_r^3)\,y^2 + (90.7/T_r-242.2/T_r^2+42.4/T_r^3) \\ & y^{(2.18+2.82/Tr)} = 0 \end{aligned} \qquad (2.9)$$

Equation (2.9) can generally be solved for y using iteration techniques. Once the value of y is found, it can then be used in equation (2.8) to solve for the value of Z.

Hall and Yarborough[14,15] pointed out that the original data values show physically unrealistic inflections for $T_r = 3.0$ and in the range of $1 < P_r < 2$. In addition, they stated that most equations of state, including the one used here, are density explicit rather than pressure explicit due to the following features:

- Finite slope of Z versus y at the critical point compared with infinite slope for Z versus P_r at the critical point (Figure 2).

- The curves have a simple form and smooth progression over the surface.
- The curves do not overlap.

Moreover, they have observed that when Z is plotted versus y for equation (2.8) and for the data taken from the Standing and Katz Z-factor chart, the plot reveals a substantial disagreement for low values of T_r. The points were found to deviate mostly at the minimum of each curve. Finally, Hall and Yarborough stated that although these observations do not establish the validity of equation (2.8), they certainly cast serious doubt upon the validity of the original values.

The authors also have compared their correlation with the modified Redlich-Kwong equation of state which was reported by Wichert and Aziz(16) to be the best overall method for predicting compressibility factors for sour Natural Gases. Data measured from twelve reservoir gas and gas-condensate systems with up to 8.5 mol % CO_2 and up to 40 mol % H_2S were used in the comparison. The calculated Z-factors were corrected for CO_2 and H_2S using the method developed by Wichert and Aziz. The results showed that the equation presented gave an overall average absolute error of 1.21 % while the modified Redlich-Kwong equation showed 2.51 % for the same data.

Finally, Hall and Yarborough pointed out that the method can be extrapolated at least to a P_r=25 and that temperature extrapolation can not be expected to be good , especially below T_r= 1.

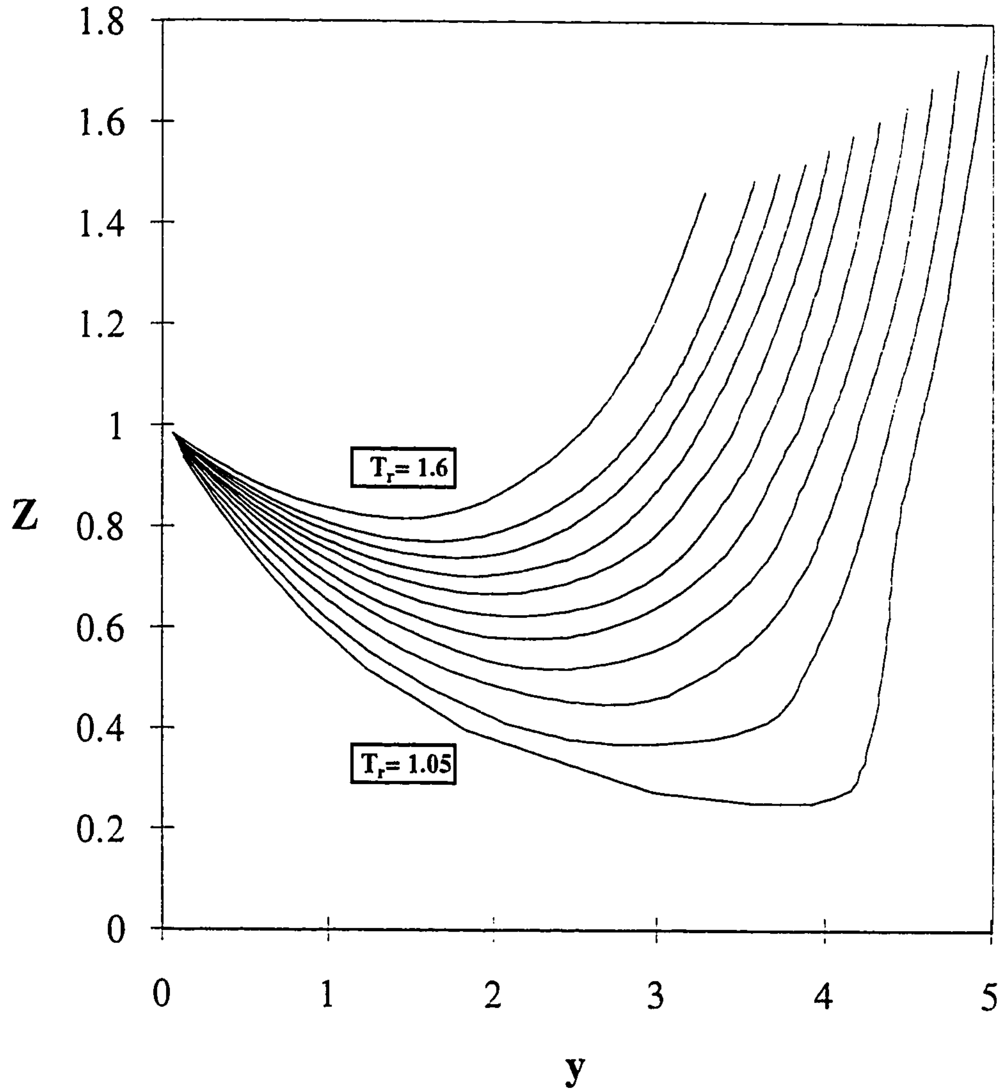

Figure 2. Z factors as a function of Hall and Yarborough gas density (y).

Comments on the Correlation:

Newton Raphson iteration technique was used to solve equation 2.8. However, among the data tested, the method failed to converge for the following nine data points: T_r= 1.05 and P_r= 3.25, 3.4, 3.75, 3.8, 4.05, 4.2, 4.3, 4.45 and 4.5. To overcome the problem, bisection method was used for those nine points.

Prior to using the bisection method, a plot of y versus P_r for T_r=1.05 was constructed in order to determine the range of y for the points failed (Figure 3). Several plots of f (y) versus y were also constructed in order to determine exactly the density root of the function for each point. The function was found to possess one real density root for each point (Figure 4). However, the function was reported to possess three or more density roots at all temperatures below the critical temperature[17]. Only the smallest and largest roots have physical significance, corresponding to vapor and liquid densities, respectively. For better understanding of both the range and shape of the gas density relation (roots of the function f(y)) a 3-D plot of y versus P_r and T_r was constructed (Figure 5).

In conclusion, although Newton Raphson iteration technique failed to converge for nine points out of 5940 points, there is no guarantee that the method will work for points in between.

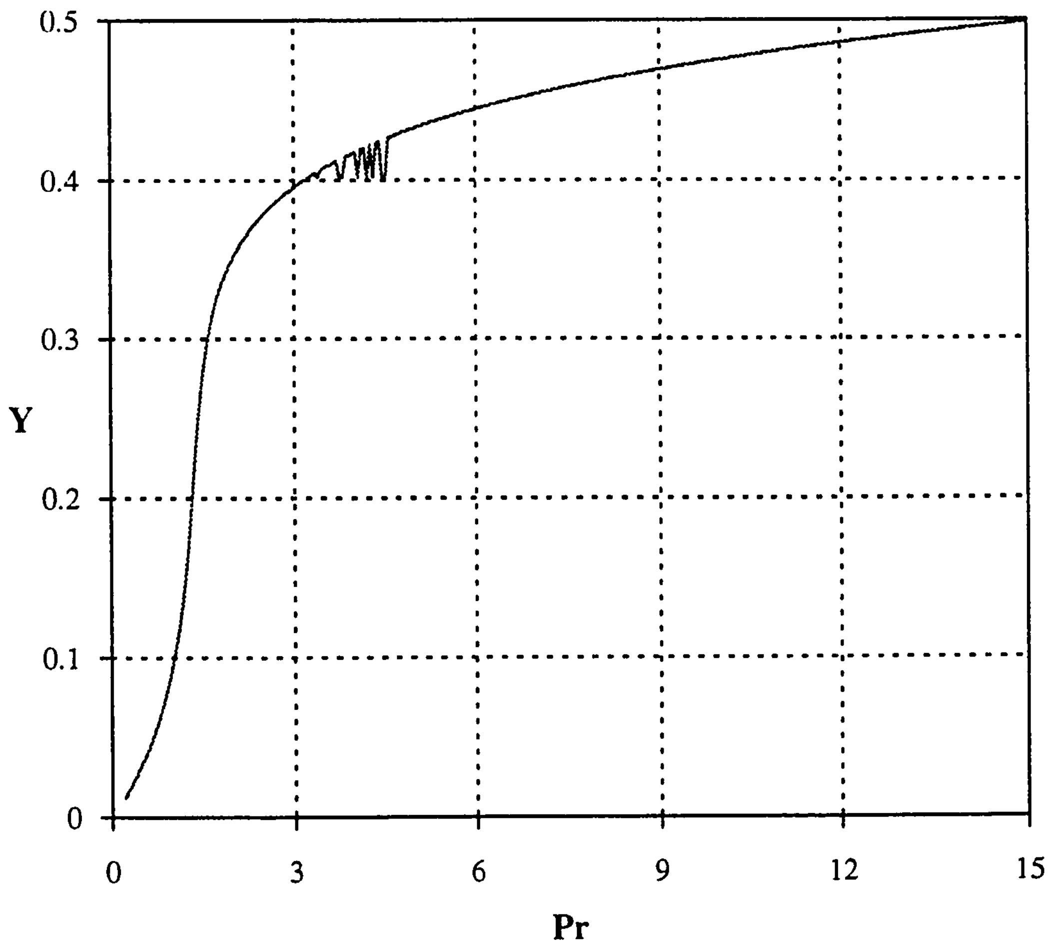

Figure 3. Hall & Yarborough gas density (y) as a function of P_r for $T_r = 1.05$.

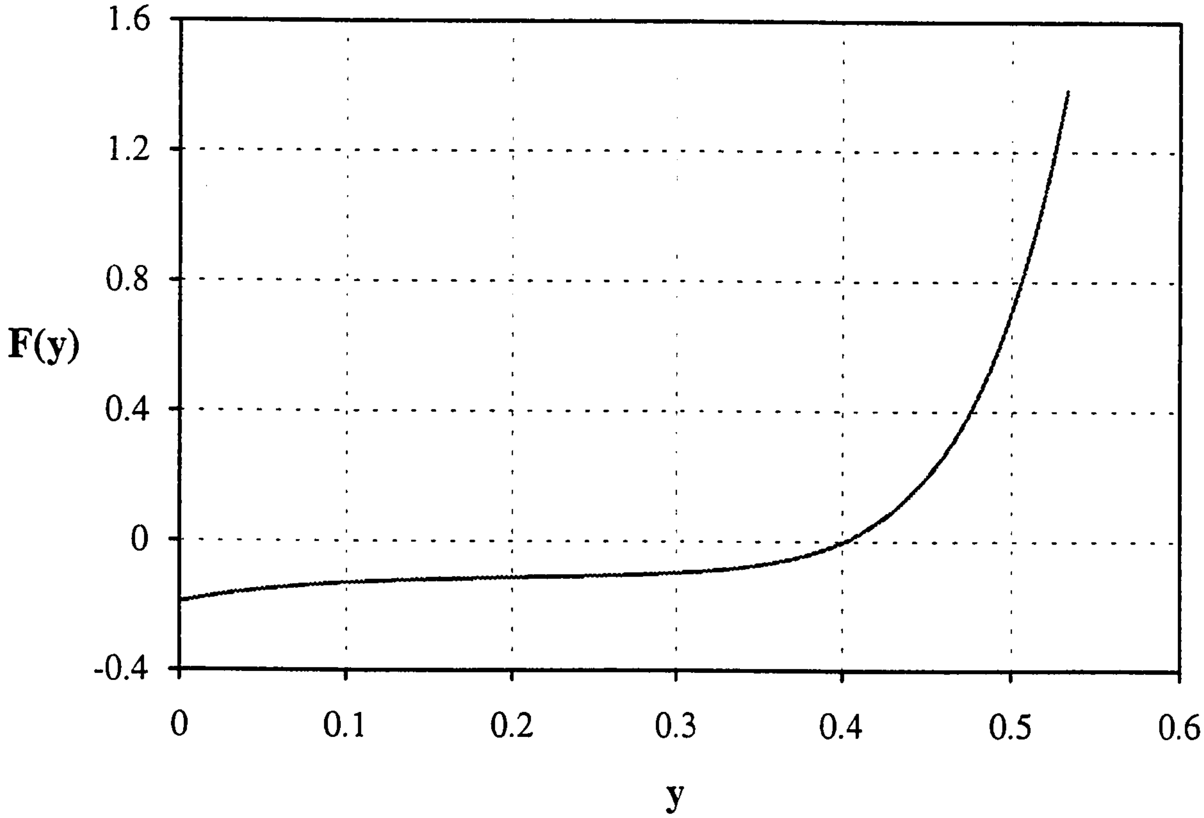

Figure 4. Hall & Yarborough gas density root estimation (i.e. P_r= 3.25 & T_r= 1.05).

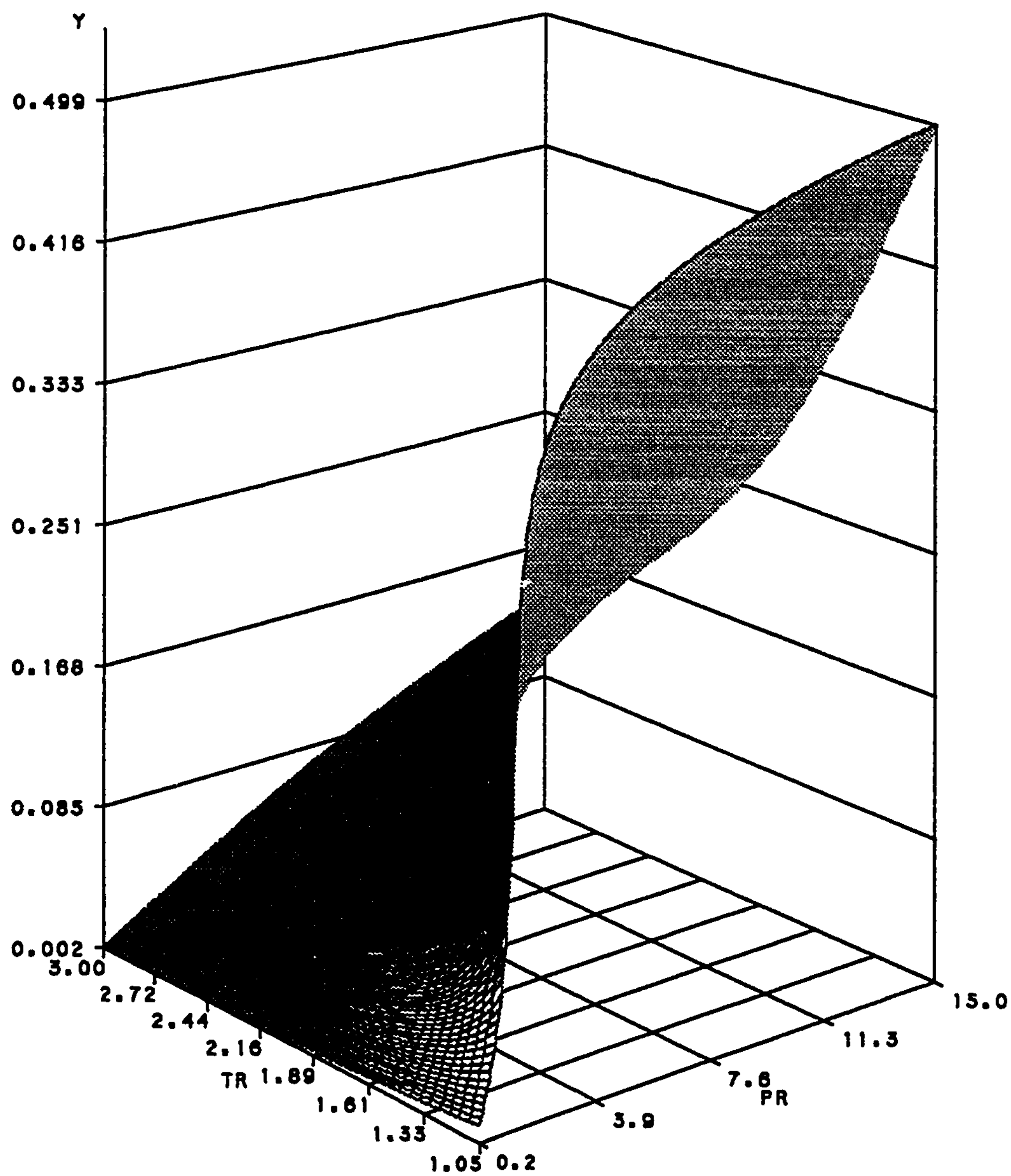

Figure 5. 3-D view of Hall & Yarborough gas density.

Dranchuk, et al. (1974)[18]. The authors developed a correlation based on the Benedict - Webb - Rubin (BWR) equation of state. The equation has eight coefficients that were determined by fitting 1500 data points taken from the Standing and Katz Z-factor chart. The proposed equation is as follows:

$$Z = 1 + (0.31506237-1.0467099/T_r-0.57832729/T_r^3)\rho_r$$
$$+ (0.53530771-0.61232032/T_r)\ \rho_r^2 + (0.61232032 * 0.10488813/T_r)\ \rho_r^5$$
$$+ 0.68157001\ (\rho_r^2/T_r^3)\ (1 + 0.68446549\ \rho_r^2)\ \exp(-0.68446549\ \rho_r^2) \qquad (2.10)$$

Where ρ_r is the pseudo reduced gas density defined by the following relation:

$$\rho_r = 0.27\ P_r\ /\ (Z\ T_r) \qquad (2.11)$$

Where 0.27 was assumed to be the compressibility factor at the critical condition. To solve equation (2.10) for Z, the left hand side of equation (2.10) is eliminated using relation (2.11) and the resulted relation is then solved for ρ_r using iterative techniques. Once ρ_r is determined, its value is then used in the reduced density relation (2.11) and the Z-factor value can be calculated.

Dranchuk, et al. Comments and Findings:

Prior to calculating the correlation coefficients, the pseudo reduced gas density was calculated using data at regular intervals from the Standing and Katz table[1]. A plot of Z as a function of ρ_r and T_r was then constructed (Figure 6). It

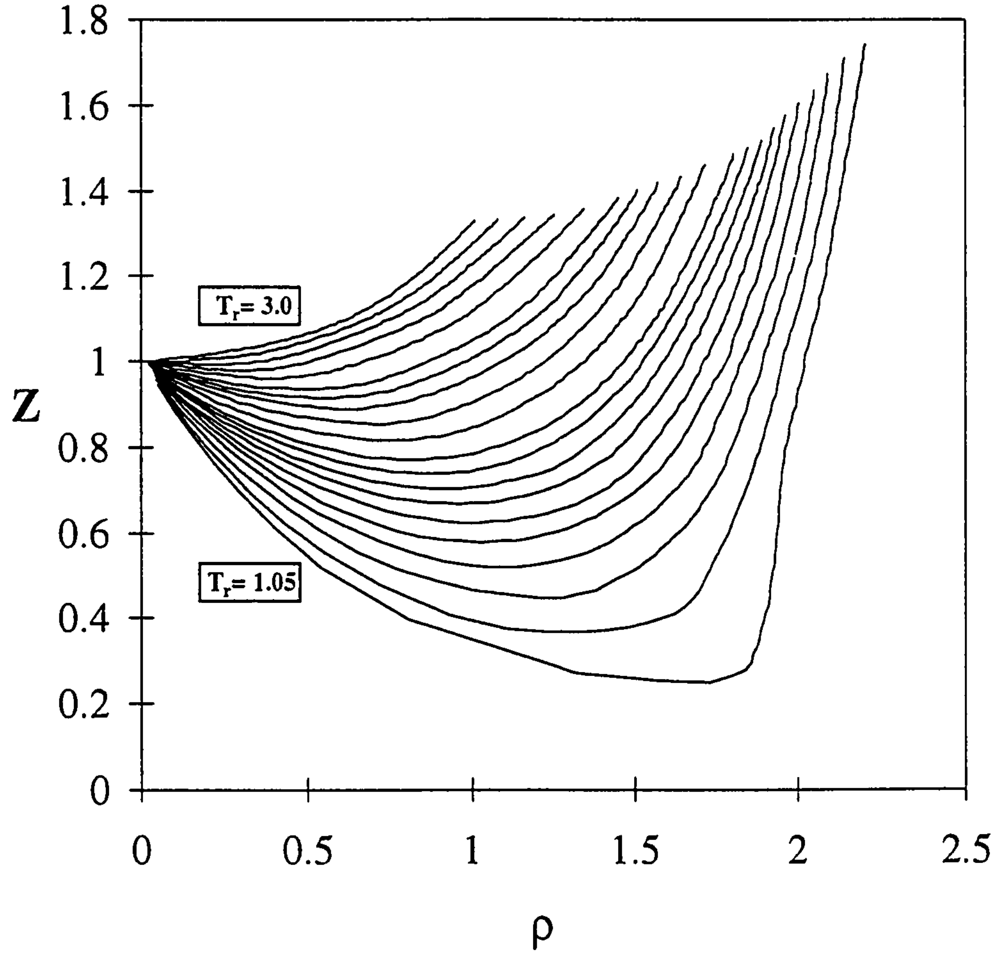

Figure 6. Standing & Katz Z factors expressed in terms of gas reduced density.

was found that the curves for $T_r \geq 1.2$ were smooth while the ones for $T_r = 1.05$, 1.10 and 1.15 have noticeable departures. These departures were in the vicinity of the critical and the authors suggested that it could be due to error in obtaining the original data or to failure of Kay's rule in correlating the data. This was further supported by the differences observed between the Standing and Katz chart and the generalized plot of compressibility factors at low reduced pressures by Brown, Katz, Oberfell and Alden[6].

Consequently, smooth continuous curves were drawn for these isotherms and smooth data were picked. The smoothed data were found later to fall between the curves of both charts. The smoothed data along with the tabulated values for the remaining isotherms were then used to find the correlation coefficients.

Although the authors proposed the eight coefficients BWR equation of state, their work actually involved fitting four equations of state:

1. The Benedict - Webb - Rubin equation of state, BWR (eight coefficients).
2. The Benedict - Webb - Rubin equation of state BWR using the $T_r = 1.05$ isotherm as a datum (Ten coefficients).
3. The Martin and Hou equation of state.
4. A variety of truncated versions of the Virial equations of state.

The studies of the Martin and Hou and the Virial equations of states indicated that closer fitting is possible but accuracy close to the other two is attainable only if the number of coefficients is increased appreciably. In addition, the ten coefficients BWR and the truncated version of the Virial equations of state could help obtain a better fit for Z than the eight coefficients BWR. The algebraic complexity in both types however is such that despite a dramatic increase in computing time and cost, the accuracy improves very slowly. In view of the nature of the original data, Dranchuk, et al. concluded that the added precision did not necessarily represent improved accuracy and that increased computing cost could not be justified at that time.

Finally, the authors compared their correlation with the Hall and Yarborough equation using 207 data points in the single phase region reported by Brown, Katz, Oberfell and Alden(6). Their results indicated a maximum error of 6.60% and 6.47% ,and a mean error of 1.15 % and 1.12% for Hall and Yarborough equation and the eight coefficients BWR equation respectively.

Comments and Observations:

The work of Hall and Yarborough ,and Dranchuk, et al. revealed some uncertainty about the original Z-factor data for low T_r values and in the vicinity of the critical . Both investigators reported some kind of departure when the data is expressed in either y or ρ_r. They however, could not justify the causes of such departure. On the other hand, they stated a possibility of experimental error in the original data itself since the departure is in the vicinity of the critical where exact experimental measurement is not easy to obtain.

This idea was also supported by the differences observed between similar charts reported by Standing and Katz, and by Brown, Katz, Oberfell and Alden. Moreover, it was found that the chart of Sage and Lacey(19) presented as a function of reduced pressure for a series of reduced temperatures for data obtained from methane, propane. n-pentane, and n-hexane is also in disagreement with the chart of Standing and Katz. Therefore to resolve such ambiguity, new experimental data within the range of uncertainty is needed.

Although the work of Dranchuk, et al. involved fitting several equations of state. only the coefficients of correlation of BWR equation of state were given. Therefore, the authors conclusions on the Martin and Hou ,and the several truncated versions of the Virial equations of states could not be verified. In addition, the authors did not state at what level the equations were truncated. Finally, Dranchuk, et al. reported that the ten coefficients BWR could be made to better fit Z-factor data than the eight coefficients BWR ,however, their error analysis results is contradicting. The reported standard error for the eight BWR is 0.87% (Maximum 7.15%) while for the ten coefficients BWR the standard error is 1.13% (Maximum 3.92%). It is clear that only the maximum error was reduced.

Dranchuk and Abou-Kassem (1975)[2]. Dranchuk and Abou-Kassem developed a correlation based on the generalized Starling equation of state. This equation has been shown[17] to be capable of predicting light hydrocarbons properties at reduced temperatures as low as $T_r = 0.3$ and reduced densities as high as $\rho_r = 3.0$.

The objective of the study was to examine the extent to which such equations could be extrapolated and to try modified fittings which would extend the region of application over the range given by Standing and Katz. The equation has eleven coefficients that were determined by fitting the 1500 data points used in the Dranchuk et al. fitting. The proposed equation has the following form:

$$Z = 1 + (0.3265 - 1.0700/T_r - 0.5339/T_r^3 + 0.01569/T_r^4 - 0.05165/T_r^5)\,\rho_r$$
$$+ (0.5475 - 0.7361/T_r + 0.1844/T_r^2)\,\rho_r^2 - 0.1056\,(-0.7361/T_r + 0.1844/T_r^2)\,\rho_r^5$$
$$+ 0.6134\,(\rho_r^2/T_r^3)\,(1 + 0.7210\,\rho_r^2)\,\exp(-0.7210\,\rho_r^2) \qquad (2.12)$$

Where ρ_r is the reduced gas density. Equation (2.12) can be solved for the compressibility factor similar to the method of Dranchuk, et al..

The authors have compared their correlation with the Hall and Yarborough and Dranchuk et al. correlations using the 1500 data points, 1350 experimental data points for gas with and without appreciable acid gas content[6,15,16], and 29 data points reported by Davis et al[20]. Results indicated that their correlation produced the least absolute error when using data from the first two sources ,however, they were very close. Their correlation gave higher error values than the other two correlations for the data reported by Davis et al.

Based on these results, the authors concluded that all three correlations yield results which are within engineering accuracy. On the other hand, their comparison of Hall and Yarborough and Dranchuk et al. correlations with experimental data showed that both are reasonably accurate when extrapolated to P_r values in excess of 20. Moreover, all three correlations were found to give large errors in the range of $T_r = 1.0$ and $P_r \geq 1.0$. Finally, the authors stated that their equation is offered as an alternative in the region $0.2 \leq P_r < 30$; $1.0 < T_r \leq 3.0$ and is recommended for the region $P_r < 1.0$; $0.7 < T_r \leq 1.0$.

CHAPTER 3

Data Acquisition

The Z-factor data used in this study were obtained from the Handbook of Natural Gas Engineering[21]. 5940 Z-factor values were initially scanned and then digitized. Prior to evaluating the correlations, the data were analyzed and verified to make sure that they are correct. The data cover the following ranges of pseudo reduced pressure and temperature: $0.2 \leq P_r \leq 15.0$ (297 P_r values) and $1.05 \leq T_r \leq 3.0$ (20 T_r values) (Appendix-A).

Z Factor Data Analysis

The data were initially plotted to check for any abnormal trends. Several points were found to be typographically in error or needed to be smoothed. Table 1 and 2 list some of the data that were corrected and smoothed respectively. A fourth digit is generally required in the Z-factor values in order to make the data smoother especially for Z-factor values at high pseudo reduced temperatures.

Table 1: Corrected Z factor data.

No.	P_r	T_r	Z(table)	Z(corrected)
1	0.25	1.80	0.998	0.988
2	1.40	1.05	0.263	0.273
3	5.65	2.20	0.004	1.004
4	7.65	1.05	0.987	0.978
5	7.65	2.20	1.965	1.065
6	14.95	1.35	1.543	1.553

Table 2: Smoothed Z factor data.

No.	P_r	T_r	Z(table)	Z(smoothed)
1	1.60	1.05	0.251	0.252
2	6.25	1.70	0.930	0.929
3	6.35	1.70	0.932	0.933
4	8.00	2.20	1.078	1.077
5	8.70	1.60	1.046	1.045
6	10.70	2.20	1.183	1.185
7	11.70	1.35	1.286	1.285
8	14.35	1.10	1.650	1.654

A plot of the data after correction and smoothing was then constructed (Figure 7). For further investigation of the shape of the function a 3-D plot of the data was constructed (Figure 8).

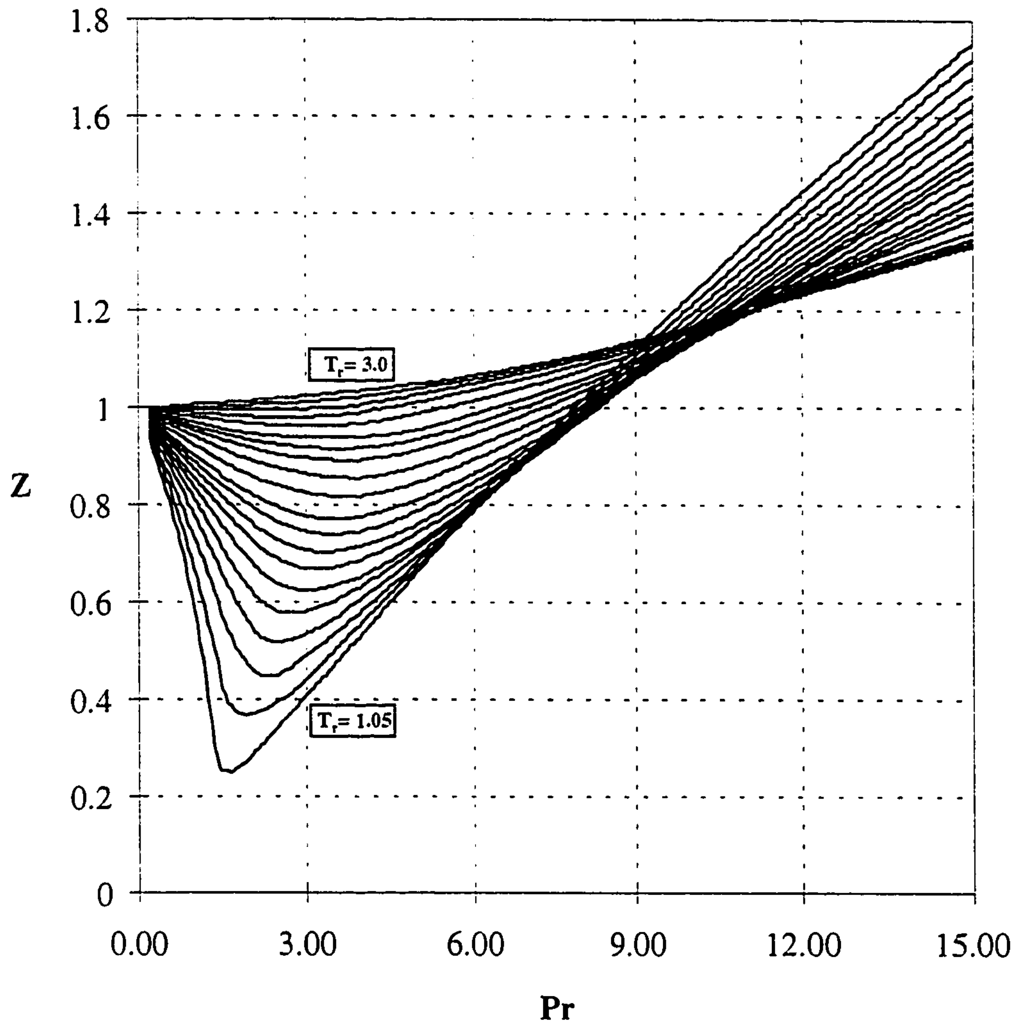

Figure 7. Regenerated Sranding & Katz Natural Gas Z factor chart.

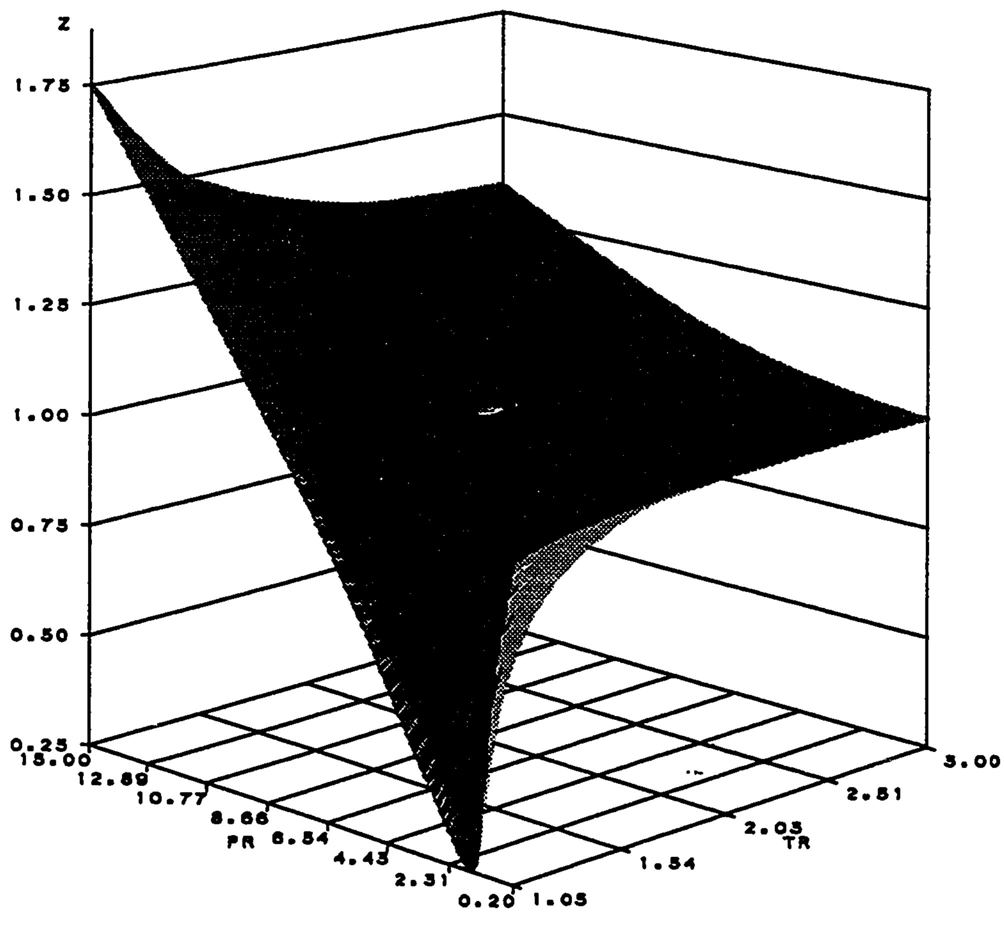

Figure 8. 3-D view of Standing & Katz Natural Gas Z factor data.

Z Factor Data Verification

Prior to evaluating the correlations verification tests were carried out to make sure that the data used is correct, smooth and consistence with the approved theories. The work of Hall and Yarborough, and Dranchuk et al. as discussed in the previous section revealed some uncertainty in the original data for T_r values of 1.05, 1.10 and 1.15. Their observation along with observations found during this research can be summarized into the following points:

- Expressing Natural Gas compressibility factors in terms of reduced gas density showed noticeable departures in the isotherms T_r = 1.05 and 1.10 with maximum departure in the T_r = 1.05 curve (Figure 6).

- Differences were observed between the Standing and Katz chart and the generalized plot of compressibility factors at low reduced pressures by Brown, et al.(6). The Standing and Katz chart curves were found to have lower values of Z than similar curves of the Brown et al. chart and especially across the minimum. The difference was substantially high in the isotherm curve of T_r = 1.05 (Figure 9).

- A similar observation was also found between the Standing and Katz chart and the chart of Sage and Lacey prepared for selected hydrocarbons in terms of reduced pressure and temperature(19) (Figure 10).

- As stated earlier, the currently used Natural Gas compressibility factor chart was constructed based on some experimental data. In addition, Methane chart was used as a guide while constructing the chart since Natural Gas was composed mainly of Methane (93 %). However, when compressibility factor chart for Methane(6) was compared with the one for Natural Gas, large differences were observed between the minimum values of Z for the lowest three isotherms T_r = 1.05, 1.10 and 1.15. These differences were substantially higher than the ones calculated for the rest of the curves with the maximum difference occurring at T_r = 1.05. The table below shows the calculated differences for the lowest ten curves.

	Methane Gas		**Natural Gas**		**Difference**
T_r	P_r	**Z(Min.)**	P_r	**Z(Min.)**	**(%)**
1.05	1.65	0.330	1.65	0.251	23.94
1.10	1.90	0.410	1.90	0.369	10.00
1.15	2.20	0.482	2.30	0.448	7.05
1.20	2.50	0.535	2.50	0.519	2.99
1.30	3.00	0.640	3.00	0.624	2.50
1.40	3.25	0.721	3.25	0.701	2.77
1.50	3.55	0.786	3.55	0.771	1.91
1.60	3.65	0.840	3.60	0.816	2.86
1.70	3.55	0.881	3.80	0.855	2.95
1.80	3.45	0.911	3.60	0.890	2.31

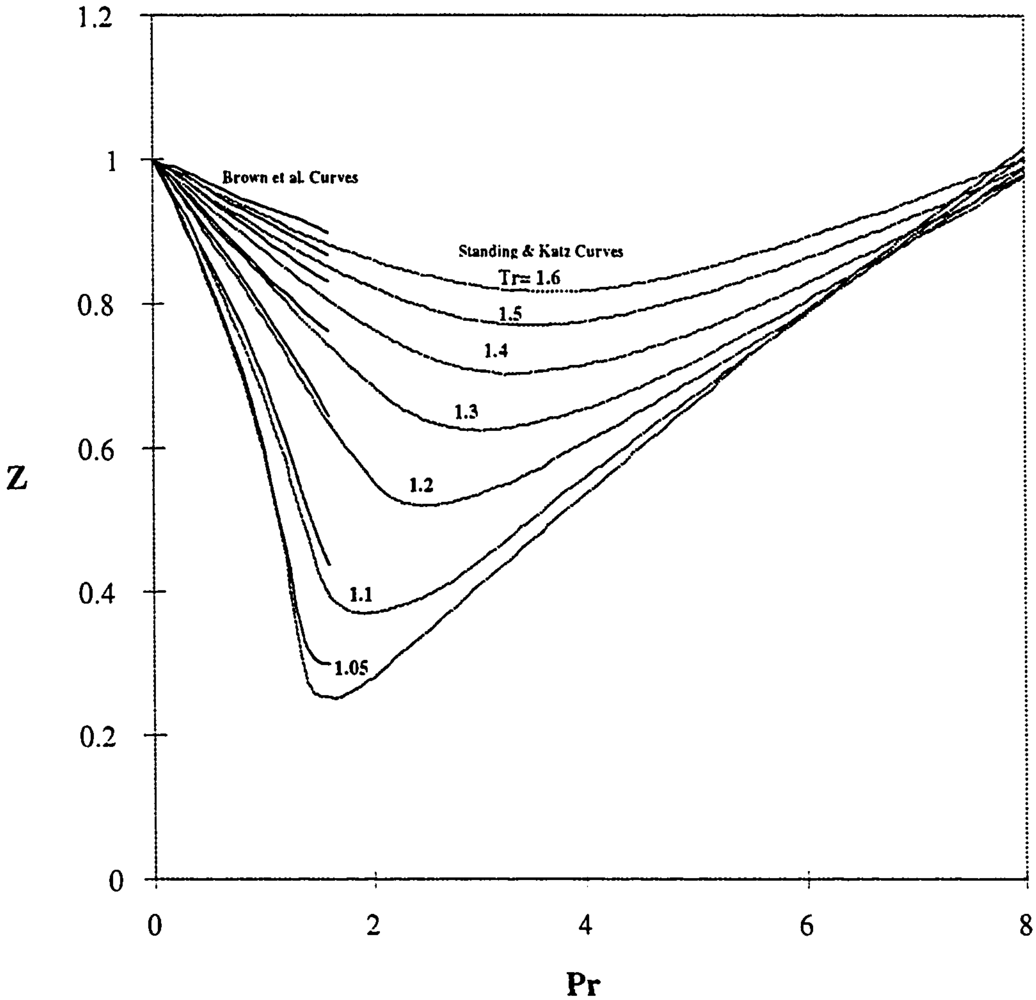

Figure 9. Combined plot of Standing & Katz, and Brown et al. charts for Natural Gas Z factors.

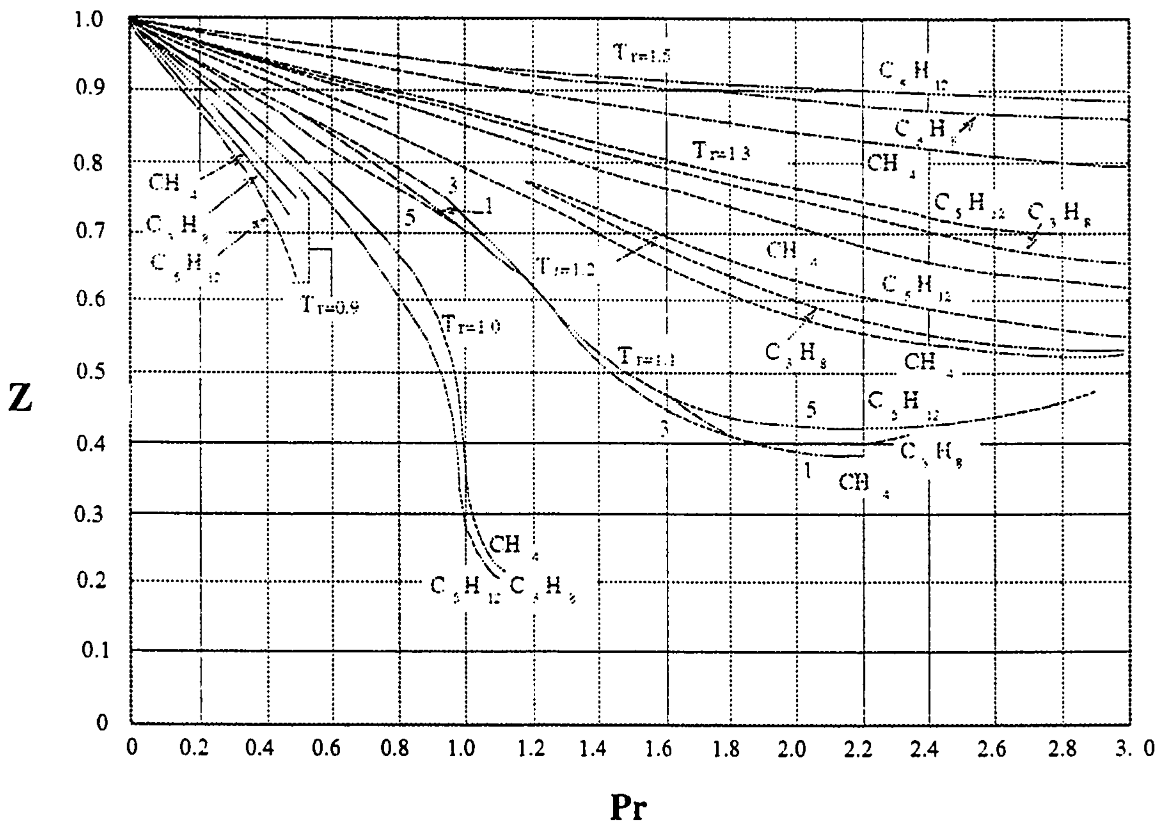

Figure 10. Sage and Lacey plot of compressibility factors for selected hydrocarbons.

Z Factor Data Smoothing

Based on the previous observations it was decided to smooth the data of the curves of T_r = 1.05 and 1.10 since they showed the highest consistence departure among the curves. Initially the Standing and Katz chart (Figure 7) was overlaid with Brown, et al.[6] generalized plot of compressibility factors at low reduced pressures (Figure 9). Using the Standing and Katz chart as a guide, the two curves for T_r = 1.05 and 1.10 of Brown, et al. plot were then extended to pseudoreduced pressures of about 5 where they met with curves of the same T_r values of the Standing and Katz chart (Figure 11).

The new curves were then digitized and a computer program was constructed to generate smoothed Z factor values at the same pseudoreduced pressures of the original data (Appendix-A). Using the new smoothed data, the Z factor chart was regenerated (Figure 12) and a plot of Z factors versus gas reduced density was also constructed (Figure 13). It is clear now that the curves of T_r = 1.05 and 1.10 show smooth progression than similar curves generated using the original data as shown in figure 6.

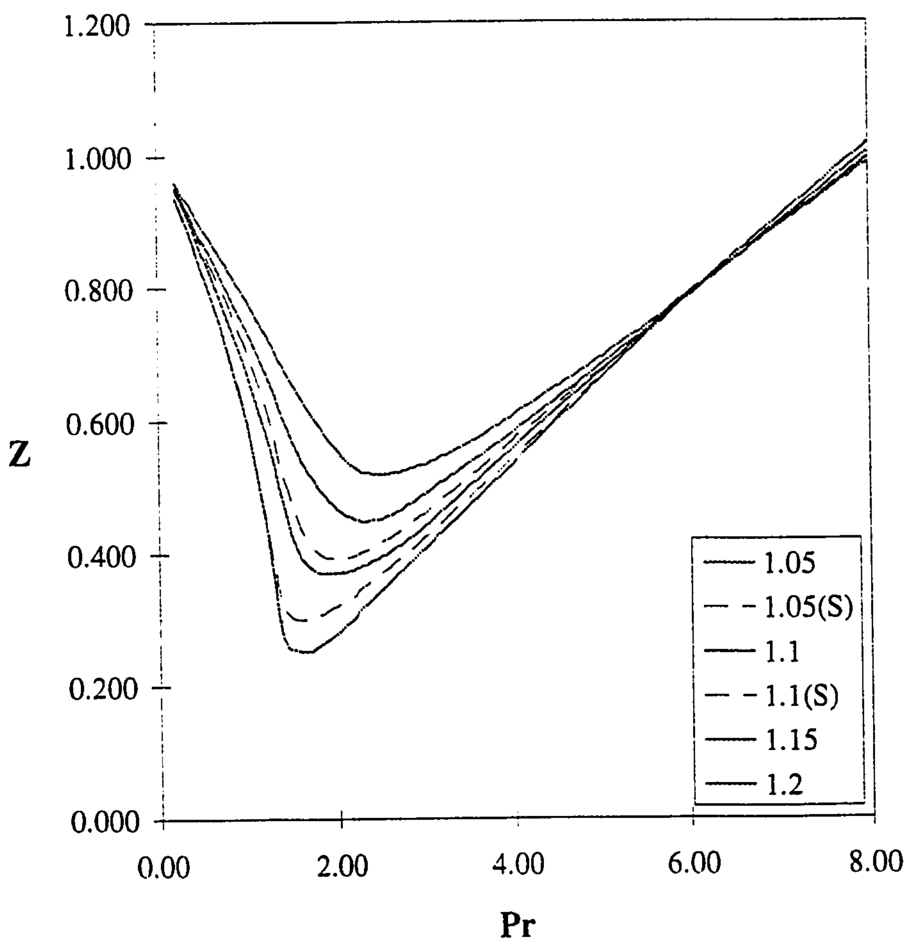

Figure 11. New smooth curves (dash lines) based on the combined plot of Standing & Katz, and Brown et al. charts for Natural Gas Z factors.

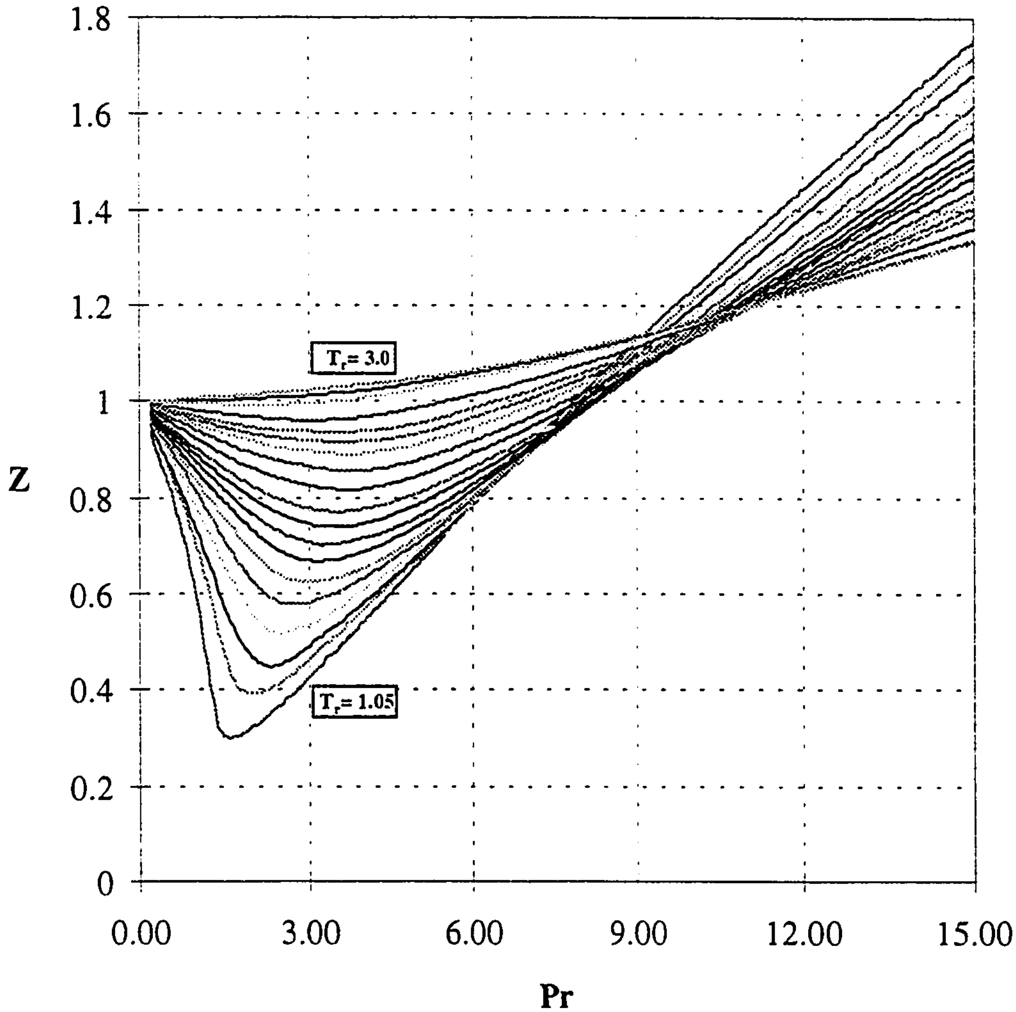

Figure 12. Regenerated Natural Gas Z factor chart based on smooth data.

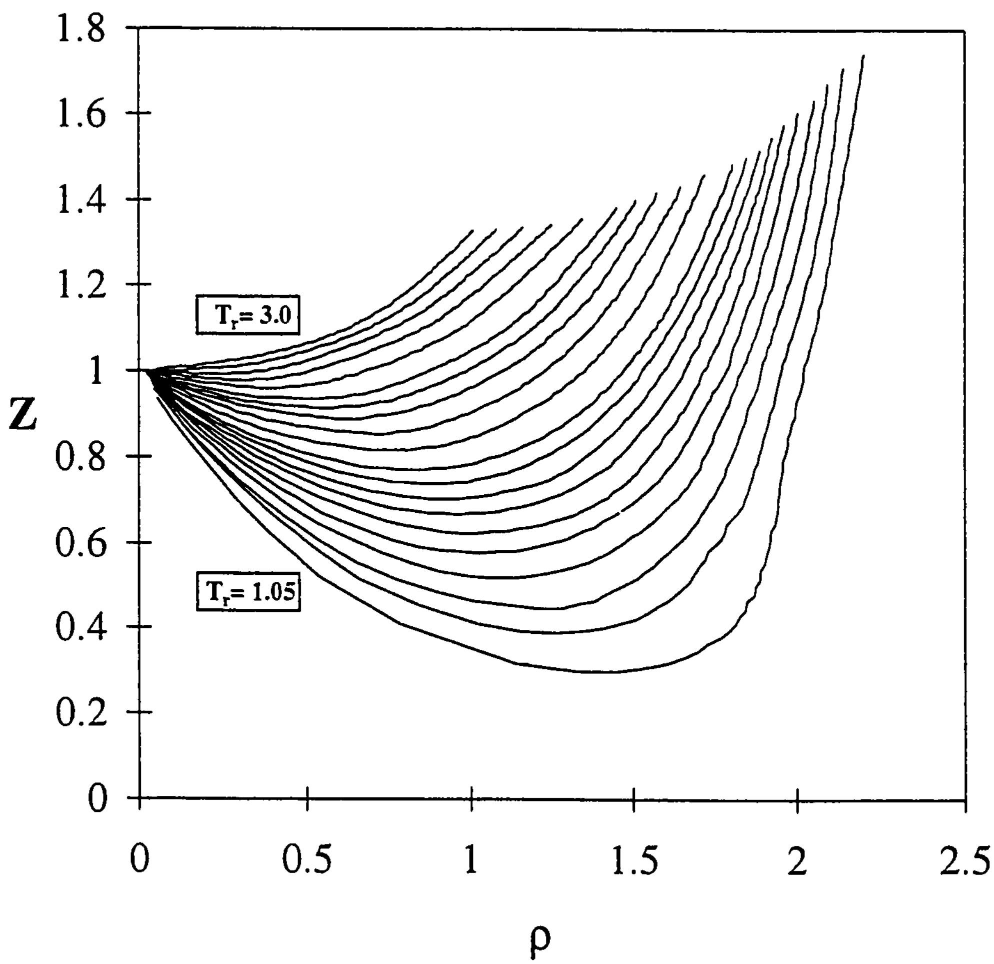

Figure 13. Standing and Katz smoothed Z factors expressed in terms of gas reduced density.

CHAPTER 4

Correlations Evaluation

The nine correlations discussed were compared using error and graphical analysis. Where each correlation permits, Z-factor values were regenerated for the whole range of pseudo reduced pressures and temperatures of the points, regardless of the limitation of each correlation. This was done to test the behavior of each correlation for the whole range of the original chart.

Error Analysis

In order to evaluate each correlation within both its valid range and beyond the valid range limits specified by the author, two types of error analyses were carried out. First, where possible the average absolute relative errors of each correlation as a function of T_r and P_r respectively were calculated for the whole range of the data and regardless of each correlation limitation. The errors were calculated in this manner to see the error behavior of each correlation. Second, the overall average absolute and arithmetic average errors, the minimum and maximum errors, and some statistical data of each correlation were calculated for different ranges of T_r and P_r. These parameters were calculated in order to compare each correlation within the limits of its valid range with the other correlations that cover a wider range. Table 3 shows a summary of each correlation.

Table 3: Z factor correlations summary sheet.

CORRELATION	Type	Number of Coefficients	Valid Range	
			T_r	P_r
Leung	Direct	16	$1.10 \leq T_r \leq 2.6$	$0.50 \leq P_r \leq 11.0$
Papay	Direct	2	N/A	N/A
Hankinson, et al.	Iterative	16	$1.10 \leq T_r \leq 3.0$	$0.40 \leq P_r \leq 15.0$
Hall and Yarborough	Iterative	10	$1.00 \leq T_r \leq 3.0$	$0.20 \leq P_r \leq 25.0$
Dranchuk, et al.	Iterative	8	$1.05 \leq T_r \leq 3.0$	$0.20 \leq P_r \leq 30.0$
D. & Abou-Kassem	Iterative	11	$1.00 \leq T_r \leq 3.0$	$0.20 \leq P_r \leq 30.0$
			$0.70 < T_r \leq 1.0$	$P_r < 1.0$
Gopal	Direct	55	$1.05 \leq T_r \leq 3.0$	$0.20 \leq P_r \leq 15.0$
Burnett	Direct	9	$1.30 \leq T_r \leq 1.85$	$P_r < P_r'$[1]
Papp	Direct	28	$1.20 \leq T_r \leq 3.0$	$0.20 \leq P_r \leq 15.0$

A. General Error Analyses:

Where correlation permits the average absolute relative errors for each isotherm using 297 values of pseudo reduced pressures and the average absolute relative errors for each isobar using 20 values of pseudo reduced temperatures were calculated. Figures 14 to 17 show plots of the data in normal and expanded scale.

[1] Refer to equation 2.5.

The plots indicated that, the Burnett correlation loses its accuracy at P_r greater than 3.65 and the correlation was found to fail to work for P_r values greater than 3.95. In terms of T_r, Burnett correlation showed relatively low error trend for T_r values greater than 1.25 because the errors were based on P_r data lower than 3.65 only. Papay's correlation showed low error values at low values of P_r and increase drastically as P_r increases. Consequently, the error of Papay's correlation with respect to T_r is high with the minimum error being around T_r=1.25. The Leung correlation has relatively low average absolute relative error for P_r less than 11.2 and T_r greater than 1.55. Gopal's correlation showed a relatively low error with oscillating trends. This might be attributed to the fact that this correlation consists of 13 different equations fitted into 13 ranges of P_r and T_r.

For a limited range of T_r, Papp's correlation gave the least error trend among the direct correlations. Papp's correlation was found to lose its accuracy tremendously when T_r is less than 1.20 and the correlation failed to work when T_r is equal to 1.10. In spite of this fact, Papp's correlation showed slight resemblance in the error trend with the iterative correlations of Hall and Yarborough, Dranchuk et, al and Dranchuk and Abou-Kassem.

Among the iterative relations, Hankinson, et al. correlation has the highest error trend. It is clear from the plots that the correlation exhibits higher error in the second range representation ($P_r \geq 5.0$) and also the error progressively increases as T_r decreases.

The other three iterative relations have relatively the same average absolute relative error trend with respect to either P_r or T_r. Generally, the error is less than 0.561 % for P_r values greater than 6.2 and less than 0.51 % for T_r values greater than 1.15. The average absolute relative error increases to a maximum of 1.976 % when P_r between 0.7 and 6.2 and to a maximum of 3.028 % when T_r is less than 1.15.

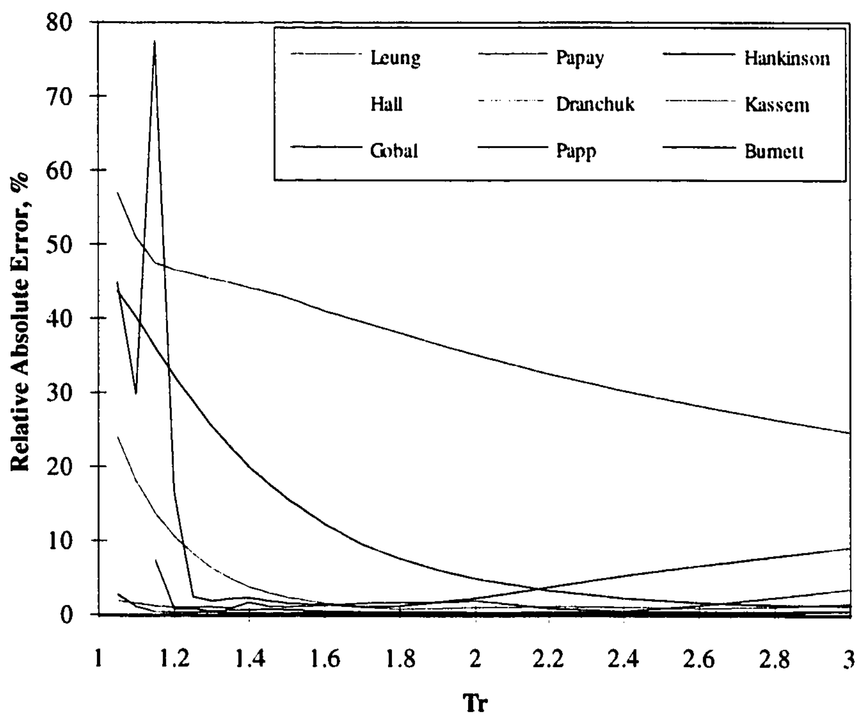

Figure 14. Statistical accuracy of Z factor correlations on isotherms.

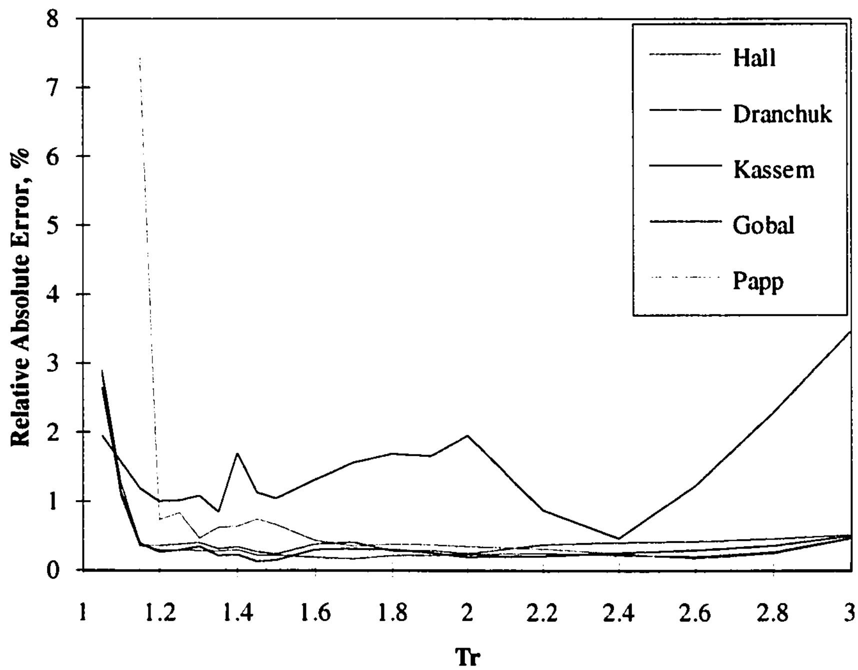

Figure 15. Statistical accuracy of Z factor correlations on isotherms (Expanded scale).

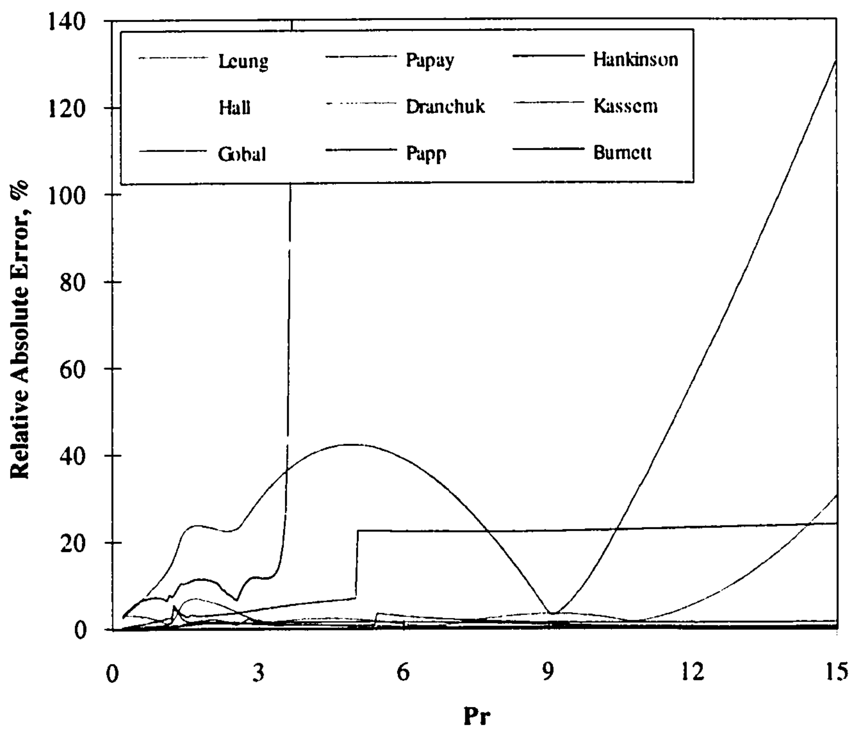

Figure 16. Statistical accuracy of Z factor correlations on isobars.

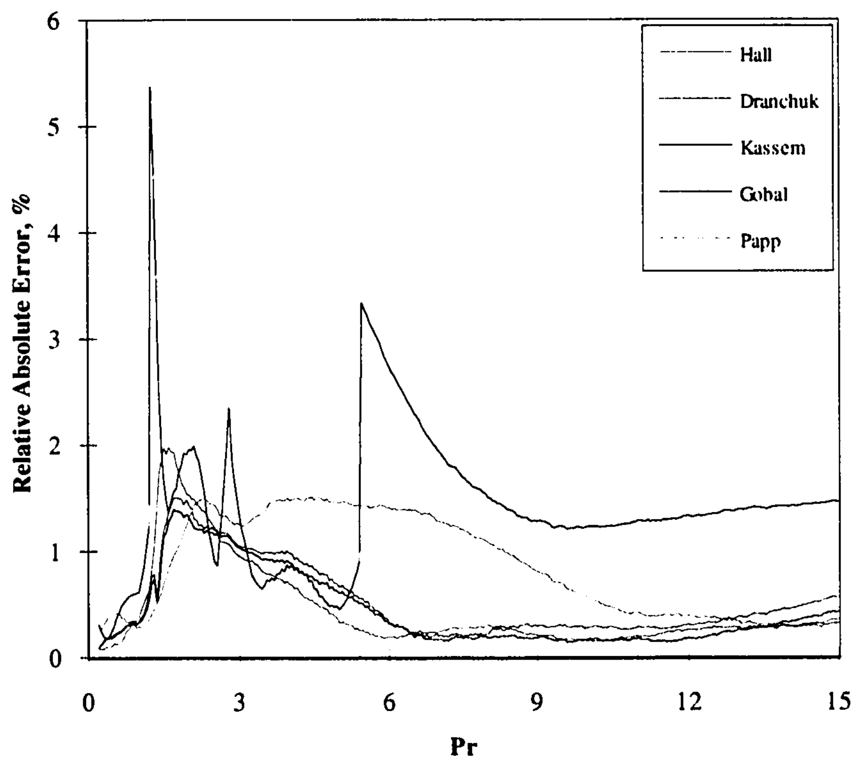

Figure 17. Statistical accuracy of Z factor correlations on isobars

(Expanded scale).

B. Specific Error Analyses:

For a fair evaluation of each correlation statistical accuracy parameters were calculated for each correlation within their specified valid ranges. Definitions of the statistical accuracy parameters are shown in Appendix-B. In order to cover the whole ranges of the nine correlations, the statistical data were calculated for five different ranges of T_r and P_r. In addition, the statistical data were also calculated based on both the original and the smoothed Z factor data. Tables 4 to 8 summarize the results based on the original data while tables 9 to 11 show similar results based on the smoothed data.

It is clear that based on the whole ranges of both original and smooth Z factor data, Papay's correlation indicated the highest average absolute relative error (Tables 4 and 9). On the other hand, Gopal's correlation showed the least average absolute relative error among the direct correlations. For a limited range of T_r, Papp's correlation gave the least average absolute relative error among the direct correlations (Table 6). Moreover, in spite of the fact that Burnett correlation has the least workable range among the nine correlations, this correlation showed the second highest average absolute relative error for data within its approximate valid range (Table 8).

In the case of iterative correlations, Hankinson et al. correlation showed the highest average absolute relative error. In fact, Hankinson et al. correlation showed the second highest error among the eight correlations working within its valid range and for both the original and smooth data (Table 5 and 10). The other three iterative correlations showed the lowest errors among the nine correlations for the five ranges analyzed and for both original and smooth data. With respect to original data the correlation of Dranchuk and Abou-Kassem indicated the lowest average absolute relative error while for the smoothed data Hall and Yarborough correlation showed the lowest average absolute relative error. Generally, most correlations showed low error values with respect to the smoothed data than the original data.

Table 4. Statistical accuracy of Z-factor correlations based on original data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 5940

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.20	15.00	7.600	4.287	0.000	-1.201
T_r	1.05	3.00	1.737	0.574	0.791	-0.536
Z-FACTOR	0.251	1.753	1.052	0.255	-0.101	0.014

STATISTICAL ACCURACY

	E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG 1964	2.291	5.292	0.000	119.958	0.203	19.342	0.361
PAPAY 1968	-8.777	40.055	0.025	169.559	0.702	66.685	-6.591
HANK 1969	16.219	16.585	0.000	57.098	0.287	27.272	-0.270
HALL 1973	-0.177	0.440	0.000	22.963	0.006	0.564	0.999
DRANCH 1974	-0.171	0.513	0.000	18.284	0.007	0.624	0.999
KASSEM 1975	-0.161	0.434	0.000	18.012	0.006	0.557	0.999
GOBAL 1977	-0.108	1.446	0.000	37.472	0.021	1.952	0.993

Table 5. Statistical accuracy of Z-factor correlations based on original data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 5567

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.40	15.00	7.700	4.229	0.000	-1.201
T_r	1.10	3.00	1.774	0.566	0.769	-0.597
Z-FACTOR	0.369	1.72	1.056	0.242	-0.046	-0.183

STATISTICAL ACCURACY

	E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG 1964	1.906	4.329	0.000	99.091	0.167	15.846	0.521
PAPAY 1968	-8.310	39.643	0.025	164.886	0.688	65.083	-7.077
HANK 1969	15.035	15.362	0.000	52.783	0.265	25.132	-0.204
HALL 1973	-0.069	0.313	0.000	11.720	0.004	0.395	1.000
DRANCH 1974	-0.042	0.394	0.000	5.514	0.005	0.478	1.000
KASSEM 1975	-0.042	0.320	0.000	5.634	0.004	0.399	1.000
GOBAL 1977	-0.062	1.436	0.000	28.716	0.021	1.961	0.993

Table 6. Statistical accuracy of Z-factor correlations based on original data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 5049

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.20	15.00	7.600	4.287	0.000	-1.201
T_r	1.20	3.00	1.850	0.550	0.711	-0.731
Z-FACTOR	0.519	1.645	1.062	0.216	0.068	-0.388

STATISTICAL ACCURACY

		E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG	1964	1.068	2.934	0.000	67.305	0.110	10.349	0.740
PAPAY	1968	-7.226	37.972	0.048	158.948	0.648	61.027	-8.039
HANK	1969	12.208	12.461	0.000	46.076	0.220	20.699	-0.040
HALL	1973	-0.009	0.250	0.000	1.964	0.003	0.318	1.000
DRANCH	1974	0.013	0.352	0.000	1.477	0.005	0.447	1.000
KASSEM	1975	0.020	0.265	0.000	1.269	0.004	0.343	1.000
GOBAL	1977	-0.020	1.425	0.000	6.899	0.021	1.971	0.991
PAPP	1979	0.131	0.469	0.000	4.733	0.006	0.535	0.999

Table 7. Statistical accuracy of Z-factor correlations based on original data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 3587

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.50	11.00	5.750	3.046	0.000	-1.201
T_r	1.10	2.60	1.641	0.436	0.745	-0.515
Z-FACTOR	0.369	1.311	0.930	0.177	-0.602	0.079

STATISTICAL ACCURACY

		E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG	1964	-0.994	1.977	0.000	37.596	0.025	2.724	0.980
PAPAY	1968	18.624	25.470	0.025	58.108	0.254	27.367	-1.057
HANK	1969	14.028	14.525	0.000	52.783	0.214	22.977	-0.450
HALL	1973	-0.148	0.344	0.000	11.720	0.004	0.446	0.999
DRANCH	1974	-0.161	0.374	0.000	5.514	0.004	0.447	0.999
KASSEM	1975	-0.157	0.320	0.000	5.634	0.004	0.396	1.000
GOBAL	1977	0.052	1.318	0.000	28.716	0.016	1.728	0.992

Table 8. Statistical accuracy of Z-factor correlations based on original data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 560

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.20	3.65	1.925	1.011	0.000	-1.207
T_r	1.30	1.80	1.513	0.164	0.442	-1.074
Z-FACTOR	0.624	0.99	0.835	0.097	-0.46	-0.803

STATISTICAL ACCURACY

		E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG	1964	-0.433	0.945	0.001	4.825	0.010	1.179	0.990
PAPAY	1968	21.724	21.724	2.492	38.031	0.192	23.017	-2.906
HANK	1969	-0.359	0.794	0.001	5.663	0.008	0.984	0.993
HALL	1973	0.098	0.175	0.000	0.672	0.002	0.214	1.000
DRANCH	1974	0.072	0.244	0.001	0.904	0.003	0.307	0.999
KASSEM	1975	0.017	0.204	0.000	0.808	0.002	0.254	1.000
GOBAL	1977	-0.230	1.339	0.002	6.800	0.015	1.807	0.976
PAPP	1979	0.415	0.626	0.003	1.916	0.006	0.716	0.996
BURNETT	1979	-1.669	1.670	0.014	4.444	0.014	1.717	0.978

Table 9. Statistical accuracy of Z-factor correlations based on smooth data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 5940

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.20	15.00	7.600	4.287	0.000	-1.201
T_r	1.05	3.00	1.737	0.574	0.791	-0.536
Z-FACTOR	0.299	1.753	1.053	0.253	-0.068	-0.061

STATISTICAL ACCURACY

		E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG	1964	2.461	5.194	0.000	119.958	0.203	19.320	0.355
PAPAY	1968	-8.605	39.947	0.046	169.559	0.701	66.647	-6.673
HANK	1969	16.330	16.658	0.000	58.455	0.287	27.271	-0.285
HALL	1973	-0.029	0.313	0.000	5.225	0.004	0.378	1.000
DRANCH	1974	-0.026	0.441	0.000	13.181	0.006	0.538	0.999
KASSEM	1975	-0.015	0.359	0.000	13.566	0.005	0.455	1.000
GOBAL	1977	0.028	1.501	0.000	39.450	0.021	1.998	0.993

Table 10. Statistical accuracy of Z-factor correlations based on smooth data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 5567

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.40	15.00	7.700	4.229	0.000	-1.201
T_r	1.10	3.00	1.774	0.566	0.769	-0.597
Z-FACTOR	0.391	1.72	1.057	0.241	-0.029	-0.218

STATISTICAL ACCURACY

		E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG	1964	1.979	4.302	0.000	99.091	0.167	15.838	0.518
PAPAY	1968	-8.243	39.616	0.048	164.886	0.688	65.064	-7.126
HANK	1969	15.092	15.381	0.000	52.957	0.266	25.131	-0.212
HALL	1973	0.000	0.267	0.000	3.449	0.004	0.334	1.000
DRANCH	1974	0.025	0.382	0.000	6.710	0.005	0.480	1.000
KASSEM	1975	0.025	0.304	0.000	5.959	0.004	0.391	1.000
GOBAL	1977	0.003	1.461	0.000	33.520	0.021	1.994	0.992

Table 11. Statistical accuracy of Z-factor correlations based on smooth data.

SUMMARY OF Z-FACTOR DATA

DATA POINTS 3587

	MIN.	*MAX.*	*AVE.*	*s*	*SKEWNESS*	*KURTOSIS*
P_r	0.50	11.00	5.750	3.046	0.000	-1.201
T_r	1.10	2.60	1.641	0.436	0.745	-0.515
Z-FACTOR	0.391	1.311	0.930	0.176	-0.571	-0.016

STATISTICAL ACCURACY

		E_r	E_a	$MIN\ E_a$	$MAX\ E_a$	E_{RMS}	*C.V.*	R^2
LEUNG	1964	-0.881	1.934	0.000	27.755	0.025	2.685	0.980
PAPAY	1968	18.727	25.428	0.048	58.322	0.254	27.358	-1.085
HANK	1969	14.116	14.556	0.000	52.957	0.214	22.979	-0.471
HALL	1973	-0.042	0.272	0.000	3.449	0.003	0.331	1.000
DRANCH	1974	-0.058	0.355	0.000	6.710	0.004	0.449	0.999
KASSEM	1975	-0.053	0.294	0.000	5.959	0.004	0.379	1.000
GOBAL	1977	0.151	1.357	0.000	33.520	0.017	1.801	0.991

Comparison Between Studies

The results of this study were compared with the results obtained by Dranchuk and Abou-Kassem[2], and Takacs[3]. It must be noted here that Dranchuk and Abou-Kassem used data from several sources in their comparison. However, in our comparison we selected the results of Dranchuk and Abou-Kassem that were based on data obtained from the same source we used. Dranchuk and Abou-Kassem calculated the overall average absolute relative error using 1500 data points and using the same points but after smoothing the data of the lowest three curves (T_r= 1.05, 1.10 and 1.15). On the other hand, Takacs study was based on 180 Z-factor values taken from Standing-Katz chart in the ranges of $1.2 \leq T_r \leq 3.0$ and $0.2 \leq P_r \leq 15.0$.

All three studies showed relatively the same overall average absolute relative error for the correlations of Hall and Yarborough, Dranchuk, et al., and Dranchuk and Abou-Kassem. In addition, all three studies showed that the best correlation is the Dranchuk and Abou-Kassem correlation. Where as this study and Takacs's study were found to agree on the correlations with the lower error values, this study showed higher error values than Takacs for the correlations with high error values like Papay, Leung and Hankinson, et al.. This might be attributed to the fact that this study used more data points in the analysis (5940 points) which are uniformly spaced and also the range of the data points is wider than Takacs's study. Finally, the Burnett correlation showed relatively low error in both studies basically due to the very limited range of the data that the correlation can work on. Table 12 summarizes the results of the three studies.

Table 12. Comparison between studies.

CORRELATIONS INVESTIGATED	Average Absolute Error, %				
	This Study		Takacs	Dranchuk & A. K.	
	Original	Smoothed		Original	Smoothed
Leung	5.292	5.194	2.115		
Papay	40.055	39.947	7.969		
Hankinson, et al.	16.585	16.658	2.799		
Hall and Yarborough	0.440	0.313	0.512	0.620	0.405
Dranchuk, et al.	0.513	0.441	0.361	0.650	0.364
Dranchuk & Abou-Kassem	0.434	0.359	0.304	0.585	0.307
Gopal	1.446	1.501	1.338		
Papp[2]	0.469	0.469	0.539		
Burnett[3]	1.670	1.670	4.601		

[2] Data of the lowest two curves T_r= 1.05 and 1.10 were not included in the analysis.

[3] This correlation works only for very limited range of the data. The data used in error calculation in both studies could be different.

Graphical Analysis

To visualize the shape of each correlation, 3-D plots were constructed. It is clear that the Papay and Burnett correlations (Figures 18 & 19) have totally different shapes from the actual function (Figure 8). On the other hand, Leung's correlation (Figure 20) showed a slight resemblance to the actual function. In general, Leung's correlation was found to fail mostly at high values of P_r and low values of T_r. Gopal's correlation (Figure 21) showed a very close shape to the actual function; however, discontinuity between the thirteen ranges of P_r and T_r is obvious. Papp's correlation was found to be the best direct correlation to resemble the actual function (Figure 22).

Among the iterative relations, the correlation of Hankinson, et al. (Figure 23) was found to describe the actual function poorly. In addition, discontinuity between the two ranges is clear. The other three iterative relations were found to describe the shape of the actual function very closely but with a different minimum Z factor value (Figures 24, 25 & 26).

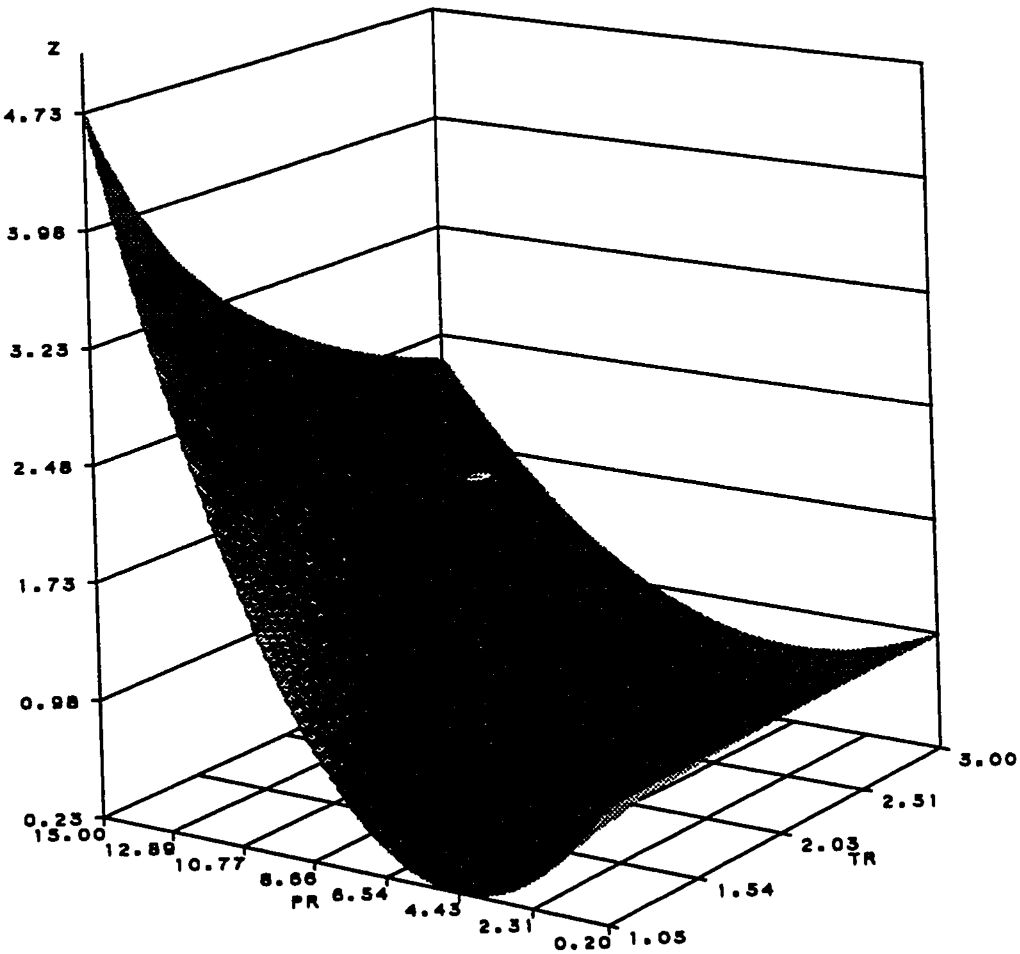

Figure 18. 3-D view of Papay correlation.

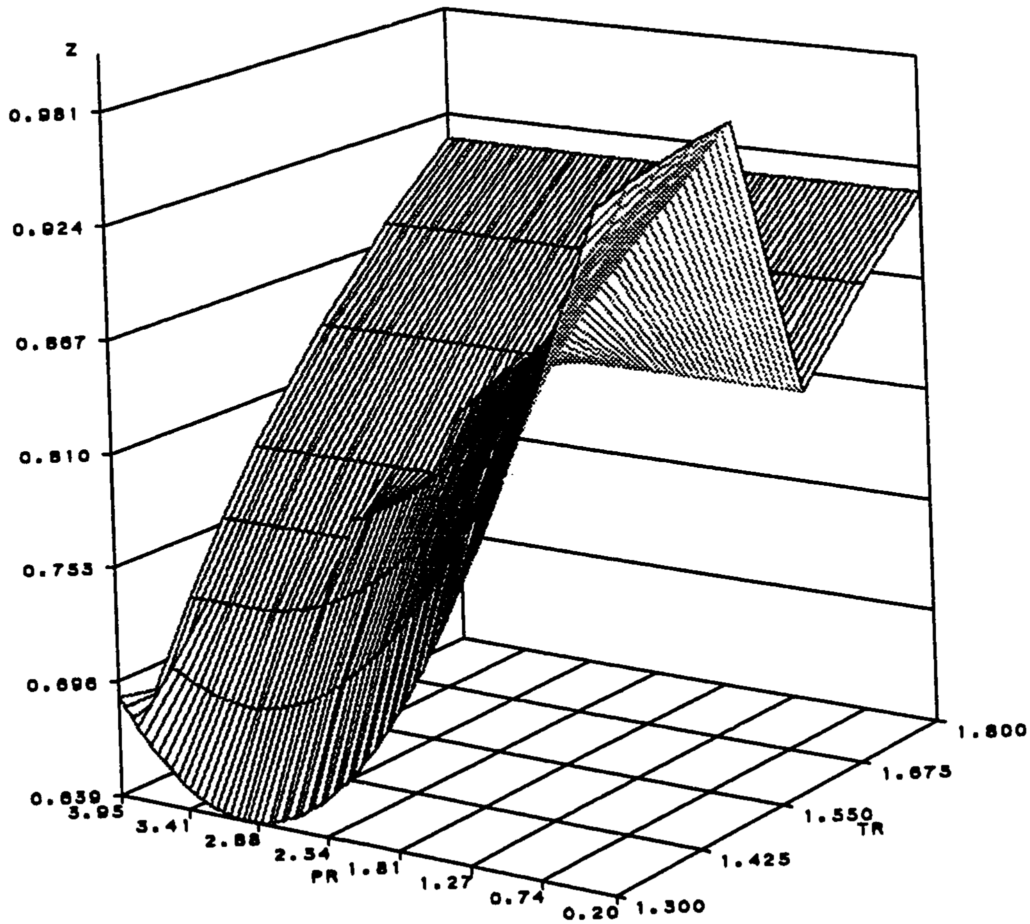

Figure 19. 3-D view of Burnett correlation.

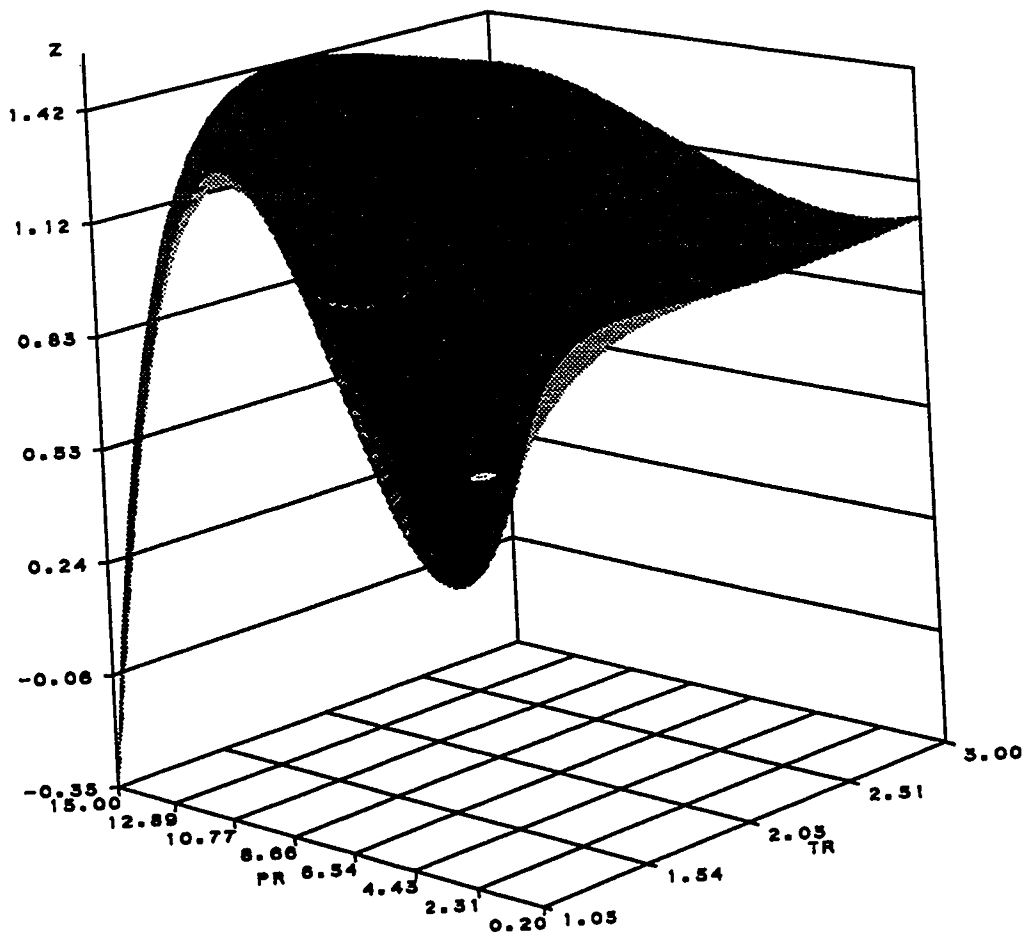

Figure 20. 3-D view of Leung correlation.

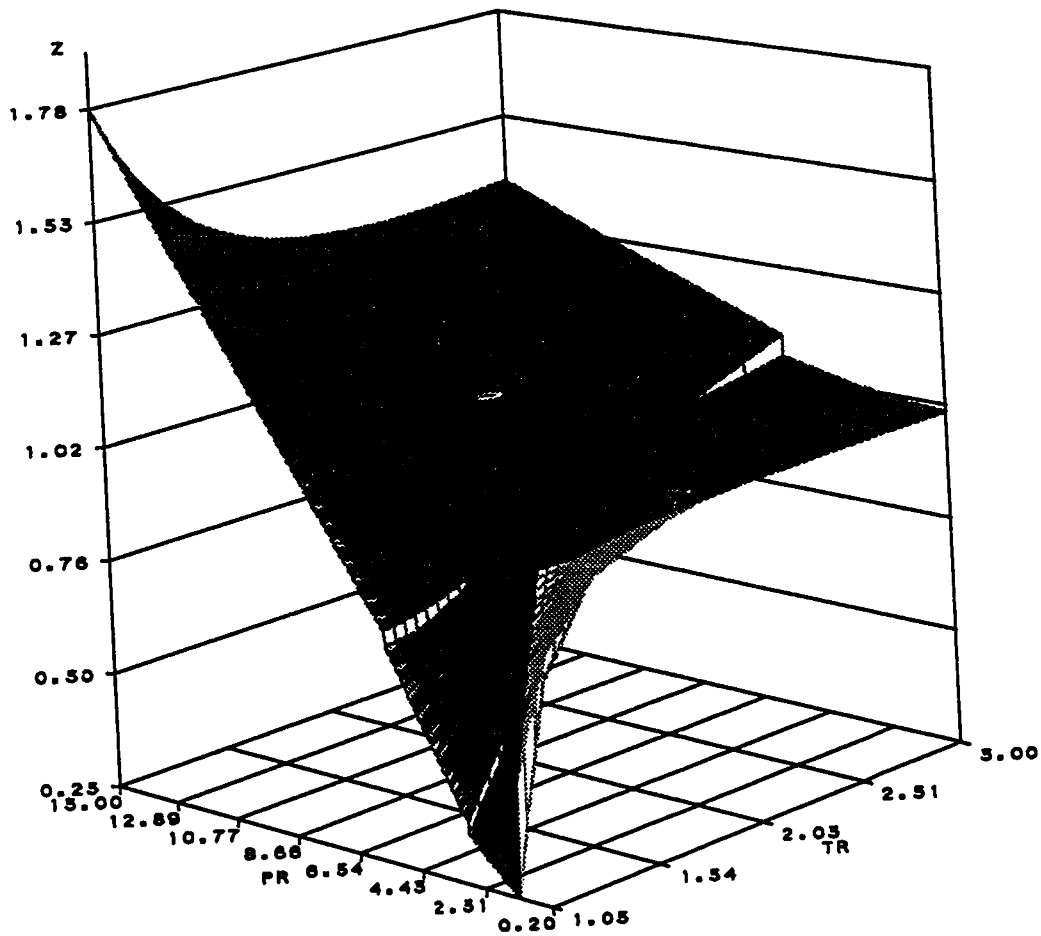

Figure 21. 3-D view of Gopal correlation.

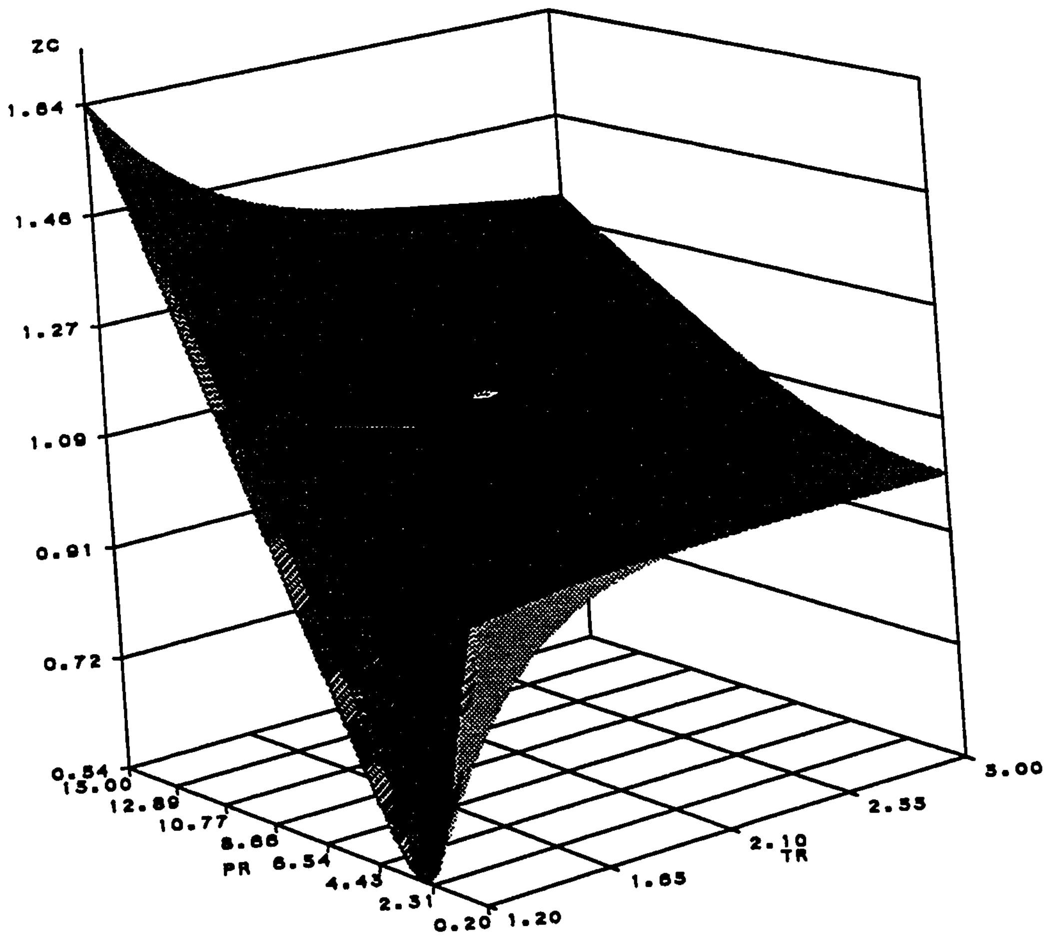

Figure 22. 3-D view of Papp correlation.

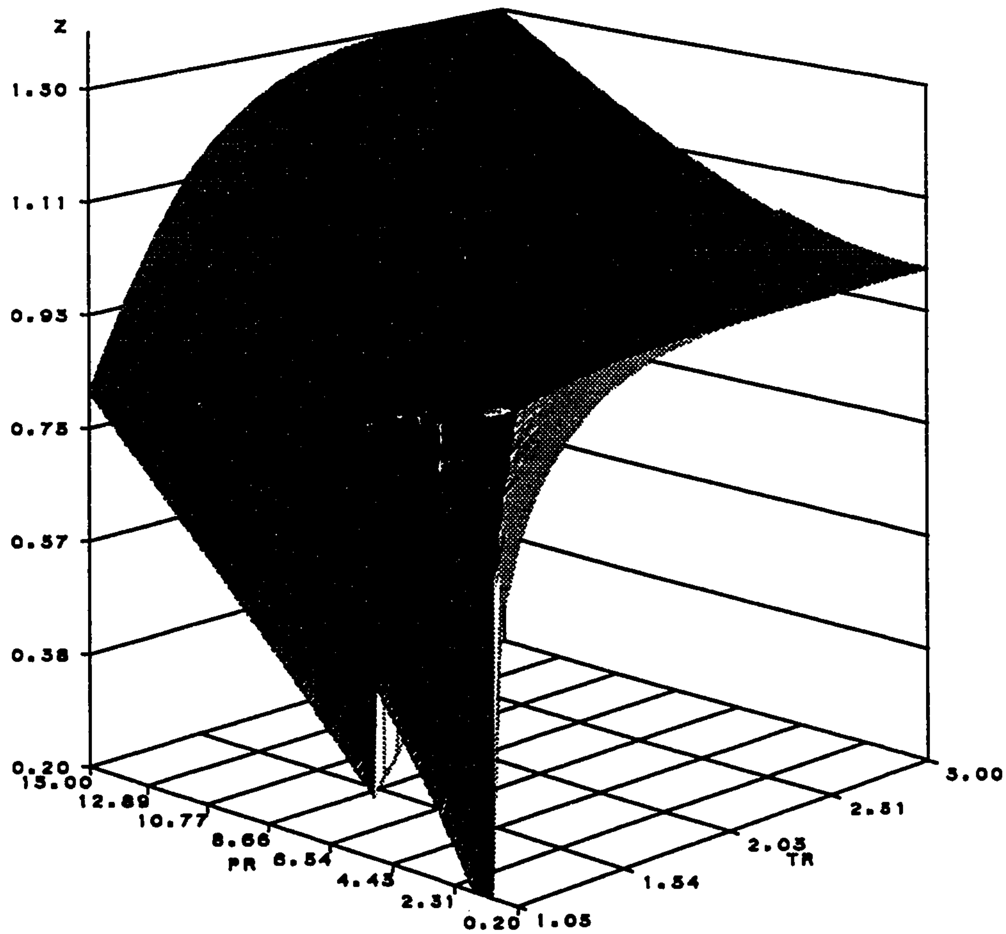

Figure 23. 3-D view of Hankinson et al. correlation.

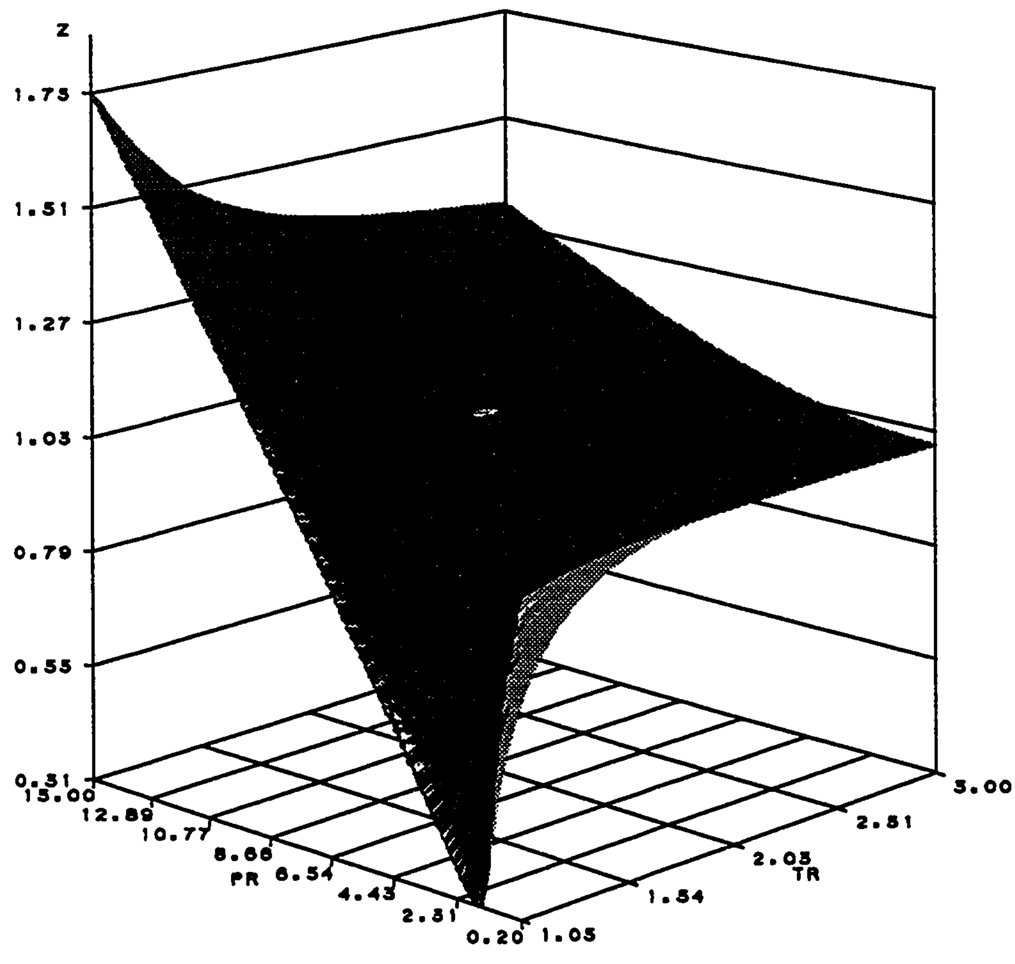

Figure 24. 3-D view of Hall & Yarborough correlation.

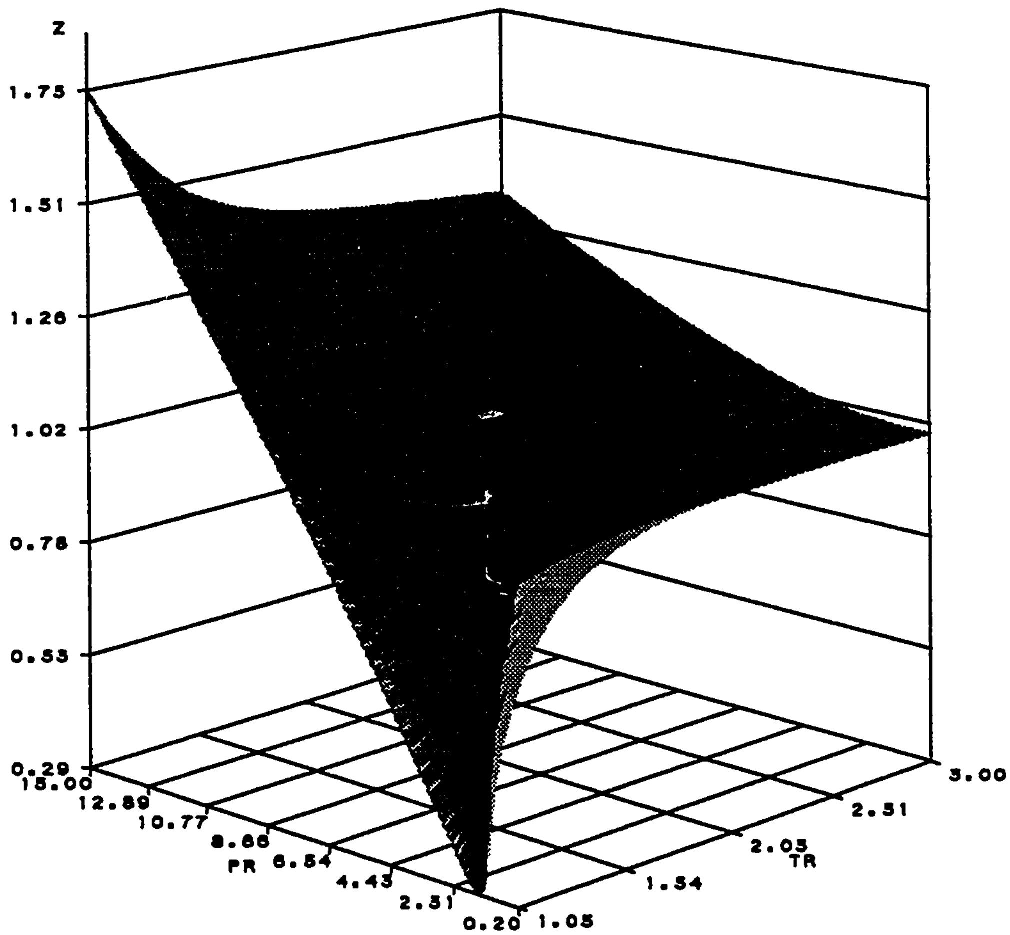

Figure 25. 3-D view of Dranchuk et al. correlation.

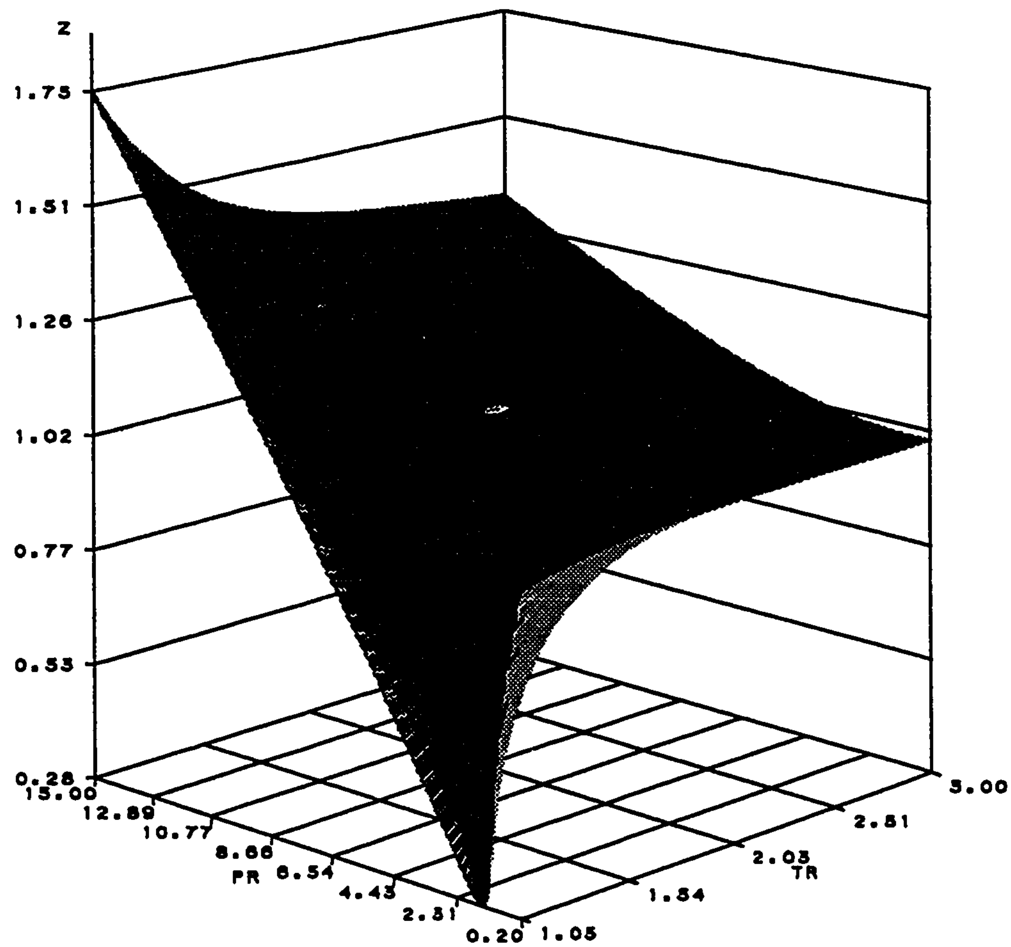

Figure 26. 3-D view of Dranchuk & Abou Kassem correlation.

`

7. Hankinson et al. correlation showed the highest average absolute relative error among the iterative correlations and came out with the second highest error among the nine correlations studied.

APPENDIXES

APPENDIX-A

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
0.20	0.938	0.934	0.948	0.952	0.953	0.959	0.964	0.969	0.971	0.973	0.978	0.979	0.984	0.987	0.990	0.991	0.993	0.995	0.998	0.999	1.000	1.001
0.25	0.922	0.918	0.934	0.939	0.941	0.949	0.955	0.960	0.964	0.968	0.972	0.974	0.980	0.984	0.988	0.990	0.992	0.995	0.998	0.999	1.000	1.001
0.30	0.905	0.901	0.920	0.926	0.928	0.938	0.946	0.951	0.956	0.962	0.965	0.969	0.976	0.981	0.985	0.988	0.990	0.994	0.997	0.999	1.000	1.001
0.35	0.887	0.883	0.905	0.912	0.914	0.927	0.937	0.943	0.949	0.956	0.959	0.964	0.972	0.978	0.983	0.986	0.989	0.994	0.997	0.999	1.000	1.001
0.40	0.868	0.865	0.889	0.898	0.900	0.916	0.928	0.934	0.942	0.949	0.953	0.959	0.968	0.975	0.980	0.983	0.987	0.992	0.996	0.998	1.000	1.002
0.45	0.849	0.849	0.872	0.884	0.887	0.905	0.919	0.926	0.934	0.943	0.948	0.954	0.964	0.972	0.978	0.981	0.985	0.991	0.995	0.998	1.000	1.002
0.50	0.829	0.831	0.855	0.868	0.874	0.894	0.909	0.918	0.926	0.937	0.942	0.949	0.960	0.969	0.976	0.979	0.983	0.990	0.994	0.998	1.000	1.002
0.55	0.810	0.810	0.839	0.850	0.862	0.883	0.900	0.909	0.919	0.930	0.937	0.944	0.956	0.966	0.974	0.978	0.982	0.989	0.994	0.998	1.000	1.002
0.60	0.790	0.789	0.822	0.834	0.849	0.872	0.890	0.900	0.912	0.923	0.931	0.939	0.952	0.963	0.971	0.976	0.981	0.988	0.993	0.997	1.000	1.003
0.65	0.769	0.769	0.804	0.818	0.835	0.861	0.881	0.893	0.905	0.917	0.926	0.935	0.949	0.961	0.969	0.974	0.980	0.987	0.993	0.997	1.000	1.003
0.70	0.747	0.750	0.785	0.803	0.820	0.850	0.871	0.885	0.898	0.911	0.921	0.930	0.945	0.958	0.967	0.972	0.978	0.986	0.992	0.997	1.001	1.004
0.75	0.722	0.726	0.767	0.789	0.806	0.839	0.861	0.878	0.890	0.905	0.916	0.925	0.942	0.955	0.965	0.971	0.977	0.985	0.992	0.997	1.001	1.004
0.80	0.698	0.701	0.748	0.772	0.791	0.827	0.851	0.870	0.883	0.899	0.910	0.920	0.938	0.952	0.962	0.969	0.975	0.984	0.991	0.997	1.001	1.005
0.85	0.672	0.677	0.729	0.755	0.776	0.815	0.842	0.861	0.876	0.893	0.905	0.916	0.935	0.950	0.960	0.967	0.974	0.983	0.991	0.997	1.001	1.005
0.90	0.645	0.653	0.710	0.737	0.761	0.802	0.832	0.852	0.870	0.887	0.899	0.911	0.931	0.947	0.958	0.965	0.972	0.983	0.990	0.996	1.001	1.006
0.95	0.618	0.624	0.690	0.719	0.748	0.791	0.823	0.845	0.863	0.881	0.895	0.906	0.927	0.944	0.956	0.963	0.971	0.982	0.990	0.996	1.001	1.006
1.00	0.590	0.595	0.670	0.701	0.734	0.780	0.813	0.838	0.856	0.875	0.890	0.901	0.923	0.941	0.953	0.961	0.970	0.981	0.989	0.996	1.001	1.007
1.05	0.555	0.560	0.647	0.683	0.719	0.768	0.803	0.829	0.849	0.869	0.885	0.897	0.920	0.939	0.951	0.960	0.969	0.980	0.989	0.996	1.001	1.007
1.10	0.520	0.524	0.624	0.663	0.703	0.755	0.793	0.820	0.842	0.862	0.879	0.893	0.917	0.936	0.949	0.958	0.967	0.979	0.988	0.996	1.002	1.008
1.15	0.482	0.487	0.602	0.643	0.688	0.743	0.783	0.812	0.836	0.856	0.874	0.889	0.913	0.933	0.947	0.957	0.966	0.979	0.988	0.996	1.002	1.008
1.20	0.444	0.449	0.580	0.619	0.672	0.731	0.773	0.804	0.829	0.850	0.869	0.884	0.909	0.930	0.945	0.955	0.964	0.978	0.987	0.996	1.002	1.008
1.25	0.398	0.411	0.555	0.595	0.656	0.719	0.763	0.797	0.823	0.845	0.864	0.880	0.905	0.928	0.943	0.954	0.963	0.978	0.987	0.996	1.002	1.008
1.30	0.351	0.375	0.530	0.567	0.640	0.707	0.753	0.789	0.817	0.839	0.859	0.876	0.901	0.925	0.941	0.952	0.962	0.977	0.987	0.996	1.002	1.009
1.35	0.307	0.346	0.505	0.542	0.623	0.695	0.743	0.781	0.810	0.834	0.855	0.872	0.898	0.923	0.940	0.951	0.961	0.976	0.986	0.996	1.002	1.009
1.40	0.273	0.317	0.480	0.518	0.605	0.682	0.732	0.772	0.803	0.828	0.850	0.868	0.895	0.920	0.938	0.949	0.960	0.975	0.986	0.996	1.002	1.010
1.45	0.259	0.310	0.453	0.495	0.588	0.670	0.721	0.765	0.797	0.824	0.846	0.864	0.892	0.918	0.936	0.947	0.959	0.975	0.986	0.996	1.002	1.010
1.50	0.254	0.302	0.425	0.475	0.570	0.657	0.710	0.757	0.791	0.817	0.841	0.859	0.889	0.915	0.934	0.946	0.957	0.974	0.985	0.995	1.003	1.010
1.55	0.253	0.300	0.409	0.454	0.556	0.646	0.701	0.750	0.785	0.811	0.837	0.855	0.886	0.913	0.932	0.945	0.956	0.973	0.985	0.995	1.003	1.010
1.60	0.252	0.299	0.393	0.435	0.541	0.634	0.691	0.742	0.779	0.805	0.832	0.851	0.882	0.910	0.930	0.943	0.954	0.972	0.984	0.995	1.003	1.011
1.65	0.251	0.300	0.386	0.421	0.529	0.623	0.682	0.735	0.773	0.800	0.828	0.847	0.880	0.908	0.929	0.942	0.953	0.972	0.984	0.995	1.003	1.011
1.70	0.252	0.301	0.378	0.411	0.517	0.612	0.672	0.728	0.767	0.795	0.823	0.843	0.877	0.905	0.927	0.941	0.952	0.971	0.983	0.995	1.004	1.012
1.75	0.256	0.304	0.374	0.404	0.507	0.602	0.664	0.720	0.761	0.790	0.819	0.840	0.874	0.903	0.925	0.940	0.951	0.971	0.983	0.995	1.004	1.012
1.80	0.260	0.306	0.370	0.399	0.497	0.592	0.656	0.712	0.755	0.784	0.815	0.837	0.871	0.900	0.923	0.938	0.950	0.970	0.983	0.995	1.004	1.012

S: Smooth data.

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
1.85	0.265	0.310	0.370	0.395	0.489	0.582	0.648	0.706	0.750	0.779	0.811	0.834	0.869	0.899	0.922	0.937	0.949	0.970	0.983	0.995	1.004	1.012
1.90	0.270	0.314	0.369	0.393	0.480	0.572	0.640	0.699	0.744	0.774	0.806	0.830	0.866	0.897	0.921	0.936	0.948	0.969	0.982	0.995	1.005	1.013
1.95	0.275	0.317	0.369	0.392	0.474	0.564	0.634	0.693	0.738	0.771	0.803	0.827	0.864	0.895	0.920	0.935	0.948	0.969	0.982	0.995	1.005	1.013
2.00	0.280	0.321	0.370	0.392	0.467	0.555	0.627	0.687	0.732	0.767	0.799	0.824	0.861	0.893	0.918	0.933	0.947	0.968	0.982	0.995	1.006	1.014
2.05	0.286	0.325	0.371	0.391	0.462	0.547	0.620	0.681	0.727	0.762	0.795	0.821	0.859	0.891	0.917	0.932	0.946	0.968	0.982	0.995	1.006	1.014
2.10	0.292	0.329	0.372	0.393	0.457	0.539	0.612	0.674	0.721	0.757	0.791	0.818	0.856	0.889	0.915	0.931	0.945	0.967	0.981	0.995	1.007	1.015
2.15	0.298	0.333	0.374	0.394	0.454	0.534	0.607	0.669	0.716	0.753	0.788	0.815	0.854	0.887	0.914	0.930	0.945	0.967	0.981	0.995	1.007	1.015
2.20	0.305	0.338	0.376	0.396	0.450	0.528	0.602	0.663	0.711	0.748	0.784	0.811	0.851	0.885	0.912	0.929	0.944	0.966	0.981	0.995	1.007	1.016
2.25	0.311	0.343	0.378	0.398	0.449	0.525	0.598	0.658	0.707	0.745	0.781	0.808	0.850	0.884	0.911	0.928	0.943	0.966	0.981	0.995	1.007	1.016
2.30	0.318	0.349	0.380	0.401	0.448	0.522	0.593	0.652	0.702	0.741	0.778	0.805	0.848	0.882	0.909	0.927	0.942	0.965	0.980	0.995	1.008	1.017
2.35	0.324	0.354	0.383	0.404	0.448	0.521	0.590	0.649	0.699	0.737	0.775	0.803	0.846	0.881	0.908	0.927	0.942	0.965	0.980	0.995	1.008	1.017
2.40	0.330	0.359	0.387	0.408	0.449	0.520	0.586	0.645	0.695	0.733	0.771	0.800	0.843	0.879	0.907	0.926	0.941	0.964	0.980	0.995	1.008	1.018
2.45	0.336	0.365	0.390	0.412	0.450	0.520	0.583	0.642	0.692	0.731	0.768	0.798	0.842	0.878	0.906	0.925	0.941	0.964	0.980	0.995	1.008	1.018
2.50	0.343	0.370	0.394	0.416	0.451	0.519	0.581	0.638	0.688	0.728	0.765	0.796	0.840	0.876	0.904	0.924	0.941	0.963	0.980	0.995	1.009	1.019
2.55	0.349	0.374	0.398	0.419	0.454	0.519	0.581	0.636	0.685	0.725	0.763	0.793	0.839	0.875	0.903	0.923	0.941	0.963	0.980	0.995	1.009	1.019
2.60	0.356	0.379	0.402	0.423	0.458	0.520	0.580	0.633	0.682	0.722	0.760	0.791	0.837	0.873	0.902	0.922	0.940	0.963	0.980	0.995	1.009	1.020
2.65	0.362	0.384	0.406	0.427	0.460	0.521	0.580	0.631	0.680	0.720	0.758	0.789	0.835	0.872	0.901	0.922	0.940	0.963	0.980	0.995	1.009	1.020
2.70	0.369	0.390	0.410	0.432	0.463	0.522	0.579	0.629	0.678	0.718	0.756	0.787	0.833	0.871	0.900	0.921	0.939	0.962	0.980	0.996	1.010	1.021
2.75	0.375	0.395	0.414	0.436	0.467	0.524	0.579	0.628	0.676	0.716	0.754	0.785	0.832	0.870	0.900	0.921	0.939	0.962	0.980	0.996	1.010	1.021
2.80	0.381	0.400	0.419	0.441	0.472	0.526	0.579	0.627	0.673	0.713	0.752	0.783	0.830	0.869	0.899	0.920	0.938	0.962	0.980	0.996	1.010	1.022
2.85	0.387	0.406	0.424	0.447	0.476	0.528	0.579	0.626	0.672	0.712	0.750	0.781	0.829	0.868	0.898	0.920	0.938	0.962	0.980	0.996	1.010	1.022
2.90	0.394	0.411	0.429	0.452	0.481	0.530	0.580	0.625	0.670	0.710	0.748	0.779	0.828	0.867	0.897	0.919	0.938	0.962	0.980	0.997	1.011	1.023
2.95	0.401	0.418	0.434	0.456	0.485	0.532	0.580	0.625	0.670	0.709	0.747	0.778	0.827	0.866	0.897	0.919	0.938	0.962	0.980	0.997	1.011	1.023
3.00	0.407	0.424	0.440	0.460	0.489	0.534	0.581	0.624	0.669	0.707	0.745	0.777	0.825	0.864	0.896	0.918	0.938	0.962	0.981	0.997	1.012	1.024
3.05	0.413	0.430	0.446	0.466	0.494	0.537	0.582	0.624	0.669	0.706	0.744	0.776	0.823	0.863	0.896	0.918	0.938	0.962	0.981	0.997	1.012	1.024
3.10	0.420	0.436	0.452	0.472	0.499	0.540	0.584	0.625	0.668	0.705	0.742	0.775	0.822	0.862	0.895	0.917	0.937	0.962	0.981	0.998	1.013	1.025
3.15	0.426	0.441	0.457	0.477	0.503	0.543	0.586	0.625	0.668	0.704	0.742	0.774	0.822	0.862	0.895	0.917	0.937	0.962	0.981	0.998	1.013	1.025
3.20	0.432	0.446	0.463	0.482	0.507	0.546	0.588	0.626	0.668	0.703	0.741	0.773	0.821	0.861	0.894	0.917	0.937	0.962	0.982	0.998	1.014	1.026
3.25	0.438	0.453	0.469	0.487	0.511	0.548	0.590	0.627	0.668	0.701	0.741	0.773	0.820	0.860	0.894	0.917	0.937	0.962	0.982	0.998	1.014	1.026
3.30	0.445	0.459	0.475	0.492	0.516	0.551	0.592	0.628	0.669	0.702	0.740	0.772	0.819	0.859	0.893	0.916	0.937	0.962	0.982	0.999	1.015	1.027
3.35	0.451	0.466	0.481	0.497	0.520	0.555	0.594	0.629	0.669	0.702	0.740	0.772	0.819	0.859	0.893	0.916	0.937	0.962	0.982	0.999	1.015	1.027
3.40	0.458	0.472	0.488	0.502	0.525	0.559	0.597	0.631	0.670	0.703	0.739	0.771	0.818	0.858	0.892	0.916	0.937	0.963	0.983	1.000	1.016	1.028
3.45	0.464	0.477	0.494	0.508	0.530	0.562	0.599	0.632	0.670	0.703	0.739	0.771	0.818	0.858	0.892	0.916	0.937	0.963	0.983	1.000	1.016	1.029

S: Smooth data.

0435

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
3.50	0.471	0.481	0.500	0.513	0.535	0.566	0.602	0.633	0.670	0.704	0.739	0.771	0.817	0.858	0.891	0.915	0.937	0.963	0.984	1.000	1.017	1.030
3.55	0.477	0.489	0.505	0.519	0.539	0.569	0.605	0.635	0.672	0.705	0.739	0.771	0.817	0.858	0.891	0.915	0.937	0.963	0.984	1.000	1.017	1.030
3.60	0.483	0.496	0.511	0.525	0.544	0.572	0.608	0.637	0.673	0.706	0.740	0.771	0.816	0.857	0.890	0.915	0.937	0.964	0.985	1.001	1.018	1.031
3.65	0.490	0.501	0.517	0.530	0.548	0.577	0.611	0.639	0.675	0.707	0.740	0.771	0.816	0.857	0.890	0.915	0.937	0.964	0.985	1.001	1.019	1.031
3.70	0.497	0.507	0.523	0.536	0.553	0.582	0.614	0.641	0.677	0.708	0.741	0.772	0.816	0.856	0.890	0.916	0.937	0.965	0.986	1.002	1.019	1.032
3.75	0.503	0.512	0.529	0.542	0.558	0.586	0.617	0.642	0.678	0.709	0.741	0.772	0.816	0.856	0.890	0.916	0.937	0.965	0.986	1.002	1.019	1.032
3.80	0.509	0.518	0.535	0.548	0.563	0.590	0.620	0.644	0.680	0.710	0.742	0.773	0.816	0.855	0.891	0.916	0.937	0.966	0.987	1.003	1.020	1.033
3.85	0.515	0.524	0.541	0.553	0.568	0.594	0.623	0.646	0.681	0.711	0.742	0.773	0.816	0.855	0.891	0.916	0.937	0.966	0.988	1.004	1.021	1.033
3.90	0.522	0.531	0.547	0.559	0.573	0.599	0.627	0.649	0.683	0.713	0.743	0.774	0.817	0.855	0.891	0.917	0.938	0.967	0.989	1.005	1.022	1.034
3.95	0.528	0.537	0.552	0.564	0.577	0.603	0.630	0.651	0.685	0.714	0.744	0.775	0.817	0.855	0.891	0.917	0.938	0.967	0.989	1.006	1.022	1.034
4.00	0.534	0.544	0.558	0.570	0.582	0.608	0.633	0.653	0.687	0.716	0.746	0.776	0.818	0.856	0.892	0.917	0.939	0.968	0.990	1.007	1.023	1.035
4.05	0.540	0.549	0.564	0.576	0.587	0.612	0.636	0.656	0.689	0.717	0.747	0.777	0.819	0.856	0.892	0.917	0.939	0.968	0.991	1.008	1.023	1.035
4.10	0.547	0.555	0.570	0.582	0.592	0.616	0.640	0.659	0.692	0.719	0.749	0.779	0.820	0.857	0.893	0.918	0.940	0.969	0.992	1.009	1.024	1.036
4.15	0.553	0.561	0.575	0.587	0.596	0.620	0.644	0.662	0.695	0.720	0.750	0.780	0.821	0.857	0.894	0.918	0.940	0.970	0.993	1.010	1.025	1.036
4.20	0.560	0.567	0.581	0.592	0.601	0.624	0.648	0.665	0.697	0.722	0.752	0.781	0.822	0.859	0.895	0.919	0.941	0.971	0.994	1.011	1.026	1.037
4.25	0.566	0.573	0.587	0.597	0.606	0.628	0.651	0.668	0.699	0.724	0.753	0.782	0.823	0.859	0.896	0.919	0.941	0.972	0.995	1.012	1.026	1.037
4.30	0.572	0.578	0.593	0.601	0.611	0.632	0.655	0.671	0.701	0.726	0.755	0.784	0.824	0.861	0.897	0.920	0.942	0.973	0.996	1.013	1.027	1.038
4.35	0.579	0.584	0.598	0.606	0.616	0.636	0.658	0.674	0.703	0.728	0.757	0.785	0.825	0.861	0.897	0.921	0.942	0.973	0.997	1.013	1.028	1.039
4.40	0.586	0.590	0.604	0.611	0.621	0.641	0.662	0.678	0.706	0.730	0.759	0.787	0.826	0.862	0.898	0.922	0.943	0.974	0.998	1.014	1.029	1.040
4.45	0.592	0.596	0.610	0.616	0.626	0.645	0.666	0.680	0.709	0.732	0.761	0.788	0.827	0.862	0.899	0.922	0.943	0.975	0.999	1.015	1.029	1.040
4.50	0.599	0.602	0.616	0.621	0.631	0.650	0.670	0.683	0.712	0.734	0.763	0.790	0.829	0.864	0.900	0.923	0.944	0.977	1.000	1.016	1.030	1.041
4.55	0.605	0.609	0.621	0.627	0.636	0.654	0.673	0.687	0.715	0.736	0.765	0.792	0.830	0.865	0.901	0.924	0.945	0.977	1.001	1.017	1.031	1.042
4.60	0.612	0.617	0.627	0.634	0.641	0.659	0.677	0.691	0.718	0.739	0.768	0.794	0.832	0.867	0.902	0.925	0.946	0.978	1.002	1.018	1.032	1.043
4.65	0.617	0.623	0.633	0.640	0.646	0.663	0.681	0.694	0.720	0.741	0.770	0.796	0.833	0.868	0.903	0.926	0.947	0.979	1.003	1.019	1.033	1.043
4.70	0.623	0.629	0.639	0.647	0.651	0.668	0.685	0.698	0.723	0.743	0.772	0.798	0.835	0.869	0.904	0.927	0.948	0.981	1.004	1.020	1.034	1.044
4.75	0.630	0.634	0.644	0.653	0.655	0.672	0.689	0.701	0.726	0.746	0.774	0.800	0.836	0.870	0.905	0.928	0.949	0.982	1.005	1.021	1.035	1.045
4.80	0.637	0.639	0.650	0.657	0.662	0.677	0.693	0.705	0.729	0.749	0.777	0.802	0.838	0.872	0.907	0.929	0.950	0.983	1.007	1.022	1.036	1.046
4.85	0.643	0.646	0.655	0.661	0.667	0.681	0.697	0.708	0.732	0.751	0.779	0.804	0.840	0.873	0.908	0.930	0.951	0.984	1.008	1.023	1.037	1.047
4.90	0.650	0.652	0.661	0.665	0.672	0.686	0.701	0.712	0.736	0.753	0.782	0.806	0.842	0.875	0.910	0.932	0.952	0.985	1.009	1.024	1.038	1.048
4.95	0.656	0.658	0.666	0.670	0.677	0.690	0.705	0.716	0.739	0.756	0.784	0.808	0.844	0.877	0.911	0.933	0.953	0.986	1.010	1.025	1.038	1.048
5.00	0.663	0.663	0.672	0.674	0.683	0.694	0.709	0.720	0.742	0.760	0.787	0.810	0.846	0.879	0.912	0.934	0.954	0.988	1.011	1.027	1.039	1.049
5.05	0.669		0.677	0.680	0.688	0.698	0.713	0.723	0.745	0.763	0.789	0.812	0.848	0.880	0.914	0.935	0.955	0.989	1.012	1.028	1.040	1.050
5.10	0.675		0.683	0.685	0.693	0.703	0.717	0.727	0.749	0.766	0.792	0.814	0.850	0.882	0.916	0.937	0.957	0.990	1.013	1.029	1.041	1.051

S: Smooth data.

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
5.15	0.681		0.689	0.690	0.698	0.708	0.721	0.731	0.752	0.769	0.795	0.817	0.851	0.884	0.917	0.938	0.958	0.991	1.014	1.030	1.042	1.051
5.20	0.688		0.695	0.696	0.703	0.713	0.725	0.735	0.755	0.772	0.798	0.820	0.853	0.886	0.919	0.940	0.960	0.992	1.016	1.031	1.043	1.052
5.25	0.694		0.700	0.701	0.708	0.717	0.729	0.739	0.758	0.775	0.800	0.822	0.855	0.887	0.920	0.941	0.961	0.993	1.017	1.032	1.044	1.052
5.30	0.701		0.706	0.706	0.713	0.722	0.734	0.743	0.762	0.778	0.803	0.825	0.858	0.889	0.921	0.943	0.963	0.995	1.018	1.033	1.045	1.053
5.35	0.707		0.712		0.718	0.726	0.738	0.747	0.765	0.781	0.806	0.827	0.860	0.891	0.923	0.944	0.964	0.996	1.019	1.034	1.046	1.053
5.40	0.713		0.718		0.723	0.731	0.742	0.751	0.769	0.785	0.809	0.830	0.862	0.893	0.925	0.946	0.966	0.998	1.021	1.035	1.047	1.055
5.45	0.720		0.723		0.728	0.735	0.746	0.755	0.772	0.788	0.812	0.832	0.865	0.895	0.926	0.947	0.967	0.999	1.022	1.036	1.048	1.056
5.50	0.727		0.729		0.733	0.740	0.751	0.759	0.776	0.792	0.815	0.835	0.868	0.897	0.928	0.949	0.969	1.000	1.023	1.038	1.019	1.057
5.55	0.733		0.734		0.738	0.745	0.755	0.763	0.779	0.795	0.818	0.837	0.870	0.899	0.929	0.950	0.970	1.001	1.024	1.039	1.050	1.058
5.60	0.739		0.740		0.744	0.750	0.760	0.768	0.783	0.799	0.821	0.840	0.872	0.900	0.931	0.952	0.972	1.003	1.026	1.040	1.051	1.058
5.65	0.745		0.745		0.749	0.755	0.764	0.772	0.787	0.802	0.824	0.843	0.874	0.902	0.933	0.953	0.974	1.004	1.027	1.041	1.052	1.059
5.70	0.751		0.751		0.754	0.760	0.769	0.777	0.791	0.806	0.827	0.846	0.877	0.905	0.935	0.955	0.976	1.006	1.028	1.042	1.053	1.060
5.75	0.757		0.756		0.759	0.764	0.773	0.781	0.795	0.809	0.830	0.848	0.879	0.907	0.937	0.957	0.977	1.007	1.029	1.043	1.054	1.061
5.80	0.763		0.762		0.764	0.769	0.778	0.785	0.799	0.813	0.833	0.851	0.882	0.909	0.939	0.959	0.979	1.008	1.031	1.045	1.055	1.062
5.85	0.769		0.768		0.770	0.774	0.782	0.789	0.803	0.817	0.836	0.854	0.884	0.911	0.940	0.961	0.980	1.009	1.032	1.046	1.056	1.063
5.90	0.775		0.774		0.777	0.779	0.787	0.794	0.807	0.821	0.839	0.857	0.887	0.913	0.942	0.963	0.982	1.011	1.033	1.048	1.057	1.064
5.95	0.781		0.779		0.782	0.784	0.792	0.798	0.810	0.824	0.842	0.860	0.889	0.915	0.944	0.965	0.984	1.012	1.034	1.049	1.058	1.065
6.00	0.788		0.785		0.787	0.790	0.797	0.803	0.814	0.828	0.845	0.863	0.892	0.918	0.946	0.967	0.986	1.013	1.036	1.050	1.059	1.066
6.05	0.794		0.790		0.792	0.795	0.801	0.807	0.818	0.831	0.848	0.865	0.894	0.920	0.948	0.968	0.987	1.015	1.037	1.051	1.060	1.067
6.10	0.800		0.796		0.797	0.800	0.805	0.812	0.822	0.835	0.852	0.868	0.897	0.922	0.950	0.970	0.989	1.017	1.038	1.052	1.061	1.068
6.15	0.805		0.801		0.802	0.805	0.809	0.816	0.826	0.838	0.855	0.870	0.899	0.924	0.952	0.971	0.991	1.018	1.039	1.053	1.062	1.069
6.20	0.811		0.807		0.808	0.810	0.814	0.820	0.830	0.842	0.858	0.873	0.902	0.927	0.954	0.973	0.993	1.019	1.041	1.055	1.063	1.070
6.25	0.816		0.812		0.813	0.815	0.818	0.824	0.834	0.846	0.861	0.876	0.904	0.929	0.956	0.975	0.995	1.020	1.042	1.056	1.064	1.071
6.30	0.822		0.818		0.818	0.820	0.823	0.829	0.839	0.850	0.864	0.880	0.907	0.931	0.958	0.977	0.997	1.022	1.043	1.057	1.065	1.072
6.35	0.829		0.824		0.823	0.825	0.828	0.833	0.843	0.853	0.867	0.883	0.909	0.933	0.960	0.979	0.998	1.023	1.045	1.058	1.066	1.072
6.40	0.835		0.830		0.829	0.830	0.833	0.838	0.848	0.857	0.871	0.886	0.912	0.936	0.962	0.981	1.000	1.025	1.047	1.059	1.067	1.073
6.45	0.841		0.835		0.834	0.835	0.837	0.842	0.852	0.861	0.874	0.889	0.915	0.938	0.964	0.983	1.001	1.026	1.048	1.060	1.068	1.074
6.50	0.848		0.841		0.839	0.840	0.842	0.847	0.856	0.865	0.878	0.892	0.918	0.941	0.966	0.985	1.003	1.028	1.049	1.062	1.069	1.075
6.55	0.853		0.846		0.844	0.845	0.846	0.851	0.860	0.868	0.881	0.895	0.920	0.943	0.968	0.987	1.005	1.029	1.050	1.063	1.070	1.076
6.60	0.858		0.852		0.850	0.851	0.851	0.855	0.864	0.871	0.884	0.898	0.923	0.945	0.970	0.989	1.007	1.031	1.052	1.064	1.072	1.077
6.65	0.864		0.857		0.855	0.856	0.856	0.859	0.868	0.875	0.888	0.900	0.925	0.947	0.972	0.991	1.008	1.032	1.053	1.065	1.073	1.078
6.70	0.870		0.863		0.860	0.861	0.861	0.863	0.872	0.880	0.892	0.903	0.928	0.950	0.974	0.993	1.010	1.034	1.054	1.067	1.074	1.080
6.75	0.875		0.868		0.865	0.866	0.865	0.867	0.876	0.884	0.895	0.906	0.930	0.952	0.976	0.995	1.011	1.036	1.055	1.068	1.075	1.081

S: Smooth data.

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
6.80	0.881		0.874		0.871	0.872	0.870	0.872	0.880	0.888	0.899	0.910	0.933	0.955	0.979	0.997	1.013	1.038	1.057	1.069	1.077	1.082
6.85	0.886		0.879		0.876	0.877	0.875	0.877	0.884	0.891	0.902	0.913	0.936	0.957	0.981	0.998	1.015	1.040	1.058	1.070	1.078	1.083
6.90	0.892		0.885		0.881	0.881	0.881	0.881	0.888	0.895	0.906	0.917	0.939	0.960	0.983	1.000	1.017	1.041	1.059	1.071	1.079	1.084
6.95	0.897		0.890		0.886	0.885	0.885	0.885	0.892	0.898	0.909	0.919	0.941	0.962	0.985	1.002	1.018	1.042	1.060	1.072	1.080	1.085
7.00	0.903		0.896		0.892	0.890	0.890	0.890	0.896	0.902	0.913	0.922	0.944	0.965	0.988	1.004	1.020	1.044	1.062	1.073	1.081	1.086
7.05	0.909		0.901		0.897	0.895	0.894	0.894	0.899	0.906	0.916	0.925	0.947	0.967	0.990	1.006	1.022	1.045	1.063	1.075	1.082	1.087
7.10	0.915		0.907		0.903	0.900	0.899	0.899	0.903	0.910	0.920	0.929	0.950	0.970	0.992	1.008	1.024	1.047	1.065	1.077	1.083	1.088
7.15	0.921		0.912		0.908	0.905	0.903	0.903	0.907	0.914	0.923	0.932	0.953	0.972	0.994	1.010	1.026	1.048	1.066	1.078	1.084	1.089
7.20	0.928		0.918		0.913	0.910	0.908	0.907	0.911	0.918	0.927	0.935	0.956	0.975	0.997	1.012	1.028	1.050	1.067	1.079	1.086	1.091
7.25	0.933		0.923		0.918	0.915	0.912	0.910	0.915	0.921	0.930	0.938	0.959	0.977	0.999	1.014	1.029	1.051	1.068	1.080	1.087	1.092
7.30	0.938		0.929		0.923	0.920	0.917	0.914	0.919	0.925	0.934	0.942	0.962	0.980	1.001	1.017	1.031	1.053	1.070	1.081	1.088	1.093
7.35	0.943		0.934		0.928	0.924	0.921	0.918	0.923	0.928	0.937	0.946	0.965	0.982	1.003	1.019	1.033	1.055	1.071	1.082	1.089	1.094
7.40	0.949		0.940		0.933	0.929	0.926	0.923	0.927	0.932	0.941	0.950	0.968	0.985	1.005	1.021	1.035	1.057	1.073	1.084	1.091	1.095
7.45	0.955		0.945		0.938	0.934	0.930	0.927	0.931	0.936	0.944	0.953	0.970	0.988	1.007	1.023	1.037	1.058	1.074	1.085	1.092	1.096
7.50	0.961		0.951		0.943	0.939	0.935	0.932	0.935	0.940	0.948	0.956	0.973	0.991	1.010	1.025	1.039	1.060	1.076	1.087	1.093	1.098
7.55	0.966		0.956		0.948	0.944	0.939	0.936	0.939	0.944	0.951	0.959	0.976	0.993	1.012	1.027	1.041	1.061	1.077	1.088	1.094	1.099
7.60	0.972		0.961		0.953	0.949	0.944	0.941	0.943	0.949	0.955	0.962	0.979	0.996	1.015	1.029	1.043	1.063	1.079	1.089	1.096	1.100
7.65	0.978		0.966		0.958	0.954	0.949	0.945	0.947	0.952	0.958	0.965	0.981	0.998	1.017	1.031	1.045	1.065	1.080	1.090	1.097	1.101
7.70	0.983		0.972		0.963	0.959	0.954	0.950	0.952	0.956	0.962	0.969	0.984	1.001	1.019	1.033	1.047	1.067	1.082	1.092	1.098	1.102
7.75	0.989		0.977		0.968	0.963	0.958	0.954	0.956	0.959	0.965	0.972	0.987	1.003	1.021	1.035	1.049	1.068	1.083	1.093	1.099	1.103
7.80	0.995		0.983		0.973	0.968	0.962	0.959	0.960	0.963	0.969	0.976	0.990	1.006	1.024	1.038	1.051	1.070	1.084	1.095	1.100	1.104
7.85	1.000		0.988		0.978	0.973	0.966	0.963	0.964	0.967	0.973	0.979	0.993	1.009	1.026	1.040	1.053	1.071	1.085	1.096	1.101	1.105
7.90	1.005		0.993		0.984	0.978	0.970	0.968	0.968	0.972	0.977	0.982	0.997	1.012	1.029	1.042	1.055	1.073	1.087	1.098	1.103	1.107
7.95	1.011		0.999		0.989	0.983	0.974	0.972	0.972	0.976	0.980	0.986	0.999	1.015	1.031	1.044	1.057	1.075	1.088	1.099	1.104	1.108
8.00	1.017		1.005		0.995	0.988	0.979	0.977	0.976	0.980	0.984	0.990	1.001	1.018	1.033	1.047	1.059	1.077	1.090	1.100	1.106	1.110
8.05	1.022		1.010		1.000	0.993	0.983	0.982	0.980	0.984	0.988	0.994	1.004	1.020	1.035	1.049	1.060	1.079	1.091	1.101	1.107	1.111
8.10	1.027		1.016		1.006	0.998	0.987	0.987	0.984	0.989	0.992	0.998	1.008	1.022	1.038	1.051	1.062	1.081	1.093	1.102	1.109	1.112
8.15	1.032		1.021		1.011	1.002	0.991	0.991	0.988	0.992	0.996	1.000	1.011	1.025	1.040	1.053	1.064	1.082	1.095	1.103	1.110	1.113
8.20	1.038		1.026		1.016	1.007	0.996	0.996	0.992	0.996	1.000	1.003	1.014	1.028	1.043	1.055	1.066	1.084	1.097	1.105	1.111	1.115
8.25	1.044		1.031		1.021	1.012	1.000	1.000	0.996	0.999	1.004	1.007	1.017	1.030	1.045	1.057	1.068	1.086	1.098	1.106	1.112	1.116
8.30	1.050		1.037		1.026	1.017	1.005	1.004	1.001	1.003	1.008	1.011	1.021	1.033	1.048	1.060	1.070	1.088	1.100	1.108	1.113	1.118
8.35	1.055		1.042		1.031	1.021	1.010	1.008	1.005	1.007	1.011	1.014	1.024	1.036	1.050	1.062	1.072	1.089	1.101	1.109	1.115	1.119
8.40	1.060		1.048		1.036	1.026	1.015	1.012	1.010	1.012	1.015	1.018	1.027	1.039	1.052	1.064	1.074	1.091	1.102	1.111	1.117	1.120

S: Smooth data.

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
8.45	1.065		1.053		1.041	1.030	1.019	1.016	1.014	1.016	1.019	1.021	1.029	1.041	1.055	1.066	1.076	1.092	1.103	1.112	1.118	1.121
8.50	1.070		1.058		1.046	1.035	1.024	1.021	1.019	1.020	1.023	1.025	1.032	1.044	1.058	1.069	1.078	1.094	1.105	1.113	1.119	1.123
8.55	1.075		1.063		1.051	1.040	1.029	1.025	1.023	1.024	1.026	1.028	1.035	1.047	1.060	1.071	1.080	1.096	1.107	1.115	1.120	1.124
8.60	1.081		1.068		1.056	1.045	1.034	1.030	1.028	1.028	1.030	1.031	1.039	1.050	1.063	1.073	1.082	1.098	1.109	1.117	1.122	1.126
8.65	1.086		1.073		1.061	1.049	1.039	1.034	1.032	1.032	1.034	1.035	1.042	1.052	1.065	1.075	1.084	1.099	1.110	1.118	1.123	1.127
8.70	1.092		1.078		1.066	1.054	1.044	1.039	1.037	1.036	1.038	1.039	1.045	1.055	1.068	1.078	1.087	1.101	1.112	1.120	1.125	1.129
8.75	1.097		1.083		1.070	1.058	1.049	1.043	1.041	1.040	1.041	1.043	1.048	1.058	1.070	1.080	1.089	1.103	1.113	1.121	1.126	1.130
8.80	1.102		1.088		1.075	1.063	1.054	1.048	1.045	1.044	1.045	1.047	1.051	1.061	1.073	1.082	1.091	1.105	1.115	1.122	1.127	1.132
8.85	1.108		1.094		1.080	1.068	1.058	1.052	1.049	1.048	1.048	1.049	1.055	1.064	1.075	1.084	1.093	1.107	1.116	1.124	1.129	1.133
8.90	1.113		1.099		1.085	1.073	1.063	1.056	1.053	1.052	1.052	1.052	1.058	1.067	1.078	1.087	1.095	1.109	1.118	1.126	1.131	1.135
8.95	1.118		1.103		1.090	1.077	1.067	1.060	1.057	1.056	1.056	1.056	1.061	1.069	1.080	1.089	1.097	1.111	1.120	1.127	1.132	1.136
9.00	1.124		1.108		1.095	1.082	1.072	1.064	1.061	1.060	1.060	1.060	1.064	1.072	1.083	1.092	1.100	1.113	1.122	1.129	1.134	1.138
9.05	1.129		1.113		1.100	1.086	1.076	1.068	1.065	1.064	1.064	1.064	1.067	1.075	1.085	1.094	1.102	1.115	1.124	1.130	1.135	1.139
9.10	1.135		1.118		1.105	1.091	1.081	1.073	1.069	1.068	1.068	1.068	1.070	1.078	1.088	1.097	1.104	1.117	1.126	1.132	1.137	1.140
9.15	1.140		1.123		1.109	1.096	1.085	1.077	1.073	1.071	1.071	1.071	1.074	1.081	1.091	1.099	1.106	1.119	1.128	1.133	1.138	1.141
9.20	1.146		1.128		1.114	1.101	1.090	1.082	1.078	1.075	1.074	1.074	1.078	1.084	1.094	1.102	1.108	1.122	1.130	1.135	1.140	1.143
9.25	1.151		1.133		1.119	1.105	1.094	1.086	1.082	1.079	1.078	1.077	1.081	1.086	1.096	1.104	1.110	1.124	1.132	1.136	1.141	1.145
9.30	1.157		1.138		1.124	1.110	1.099	1.091	1.086	1.083	1.082	1.081	1.081	1.089	1.099	1.106	1.112	1.126	1.134	1.138	1.143	1.147
9.35	1.162		1.143		1.129	1.115	1.104	1.095	1.090	1.087	1.086	1.084	1.087	1.092	1.101	1.108	1.114	1.128	1.136	1.140	1.145	1.148
9.40	1.167		1.148		1.134	1.120	1.109	1.100	1.095	1.092	1.090	1.088	1.090	1.095	1.104	1.111	1.117	1.130	1.138	1.142	1.147	1.149
9.45	1.172		1.153		1.139	1.124	1.113	1.104	1.099	1.096	1.094	1.092	1.093	1.098	1.106	1.113	1.119	1.132	1.140	1.143	1.148	1.150
9.50	1.178		1.159		1.144	1.129	1.118	1.109	1.103	1.100	1.098	1.096	1.097	1.102	1.109	1.116	1.122	1.134	1.142	1.145	1.150	1.152
9.55	1.182		1.164		1.149	1.134	1.122	1.113	1.107	1.104	1.101	1.099	1.100	1.104	1.111	1.118	1.124	1.136	1.143	1.147	1.151	1.153
9.60	1.189		1.169		1.154	1.139	1.127	1.118	1.111	1.108	1.104	1.102	1.103	1.107	1.114	1.121	1.126	1.139	1.145	1.149	1.153	1.155
9.65	1.194		1.174		1.158	1.143	1.132	1.122	1.115	1.112	1.108	1.105	1.106	1.110	1.117	1.123	1.128	1.141	1.147	1.150	1.154	1.156
9.70	1.200		1.179		1.163	1.148	1.137	1.127	1.120	1.116	1.112	1.109	1.109	1.113	1.120	1.126	1.131	1.143	1.149	1.152	1.156	1.158
9.75	1.205		1.184		1.168	1.153	1.141	1.131	1.124	1.120	1.115	1.113	1.113	1.116	1.122	1.128	1.133	1.145	1.150	1.154	1.157	1.159
9.80	1.210		1.189		1.173	1.158	1.146	1.135	1.128	1.124	1.119	1.117	1.117	1.119	1.125	1.131	1.136	1.148	1.152	1.156	1.159	1.161
9.85	1.215		1.194		1.178	1.162	1.150	1.139	1.132	1.128	1.123	1.120	1.120	1.122	1.127	1.133	1.138	1.150	1.154	1.157	1.160	1.162
9.90	1.221		1.200		1.183	1.167	1.155	1.143	1.137	1.132	1.127	1.123	1.123	1.126	1.130	1.135	1.140	1.152	1.156	1.159	1.162	1.164
9.95	1.226		1.205		1.187	1.171	1.159	1.147	1.141	1.136	1.130	1.126	1.126	1.129	1.133	1.138	1.142	1.154	1.158	1.161	1.163	1.165
10.00	1.231		1.210		1.192	1.176	1.164	1.152	1.145	1.140	1.134	1.130	1.130	1.132	1.136	1.141	1.144	1.156	1.160	1.163	1.165	1.167
10.05	1.236		1.215		1.197	1.180	1.168	1.156	1.149	1.144	1.138	1.134	1.133	1.135	1.139	1.143	1.146	1.158	1.162	1.164	1.166	1.168

S: Smooth data.

Standing and Katz Natural Gas Z Factor Data

Pr\Tr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
10.10	1.242		1.220		1.202	1.185	1.173	1.160	1.153	1.148	1.142	1.138	1.137	1.138	1.141	1.145	1.149	1.160	1.164	1.166	1.168	1.170
10.15	1.247		1.225		1.206	1.190	1.177	1.164	1.157	1.152	1.145	1.141	1.140	1.141	1.144	1.147	1.151	1.162	1.165	1.167	1.169	1.171
10.20	1.252		1.230		1.211	1.195	1.181	1.169	1.162	1.155	1.149	1.145	1.143	1.144	1.147	1.150	1.154	1.164	1.167	1.169	1.171	1.173
10.25	1.257		1.235		1.216	1.199	1.185	1.174	1.166	1.159	1.153	1.148	1.146	1.147	1.149	1.152	1.156	1.166	1.169	1.171	1.173	1.175
10.30	1.263		1.240		1.221	1.204	1.190	1.179	1.170	1.163	1.157	1.152	1.149	1.150	1.152	1.155	1.159	1.168	1.171	1.173	1.175	1.177
10.35	1.268		1.245		1.226	1.208	1.194	1.183	1.174	1.167	1.160	1.156	1.152	1.153	1.155	1.157	1.161	1.170	1.173	1.174	1.176	1.178
10.40	1.274		1.250		1.231	1.213	1.199	1.188	1.178	1.171	1.164	1.160	1.156	1.156	1.158	1.160	1.164	1.172	1.175	1.176	1.178	1.180
10.45	1.279		1.255		1.236	1.218	1.203	1.192	1.182	1.175	1.168	1.164	1.159	1.159	1.160	1.162	1.166	1.174	1.177	1.177	1.179	1.181
10.50	1.285		1.260		1.241	1.223	1.208	1.196	1.186	1.179	1.172	1.168	1.162	1.162	1.162	1.165	1.169	1.176	1.179	1.179	1.181	1.183
10.55	1.290		1.265		1.246	1.227	1.212	1.200	1.190	1.183	1.176	1.171	1.165	1.165	1.165	1.168	1.171	1.178	1.181	1.181	1.182	1.184
10.60	1.296		1.270		1.251	1.232	1.217	1.204	1.195	1.187	1.180	1.174	1.169	1.168	1.168	1.171	1.174	1.181	1.183	1.183	1.184	1.186
10.65	1.301		1.275		1.256	1.236	1.221	1.208	1.199	1.191	1.183	1.177	1.172	1.170	1.171	1.174	1.176	1.183	1.184	1.184	1.185	1.187
10.70	1.307		1.280		1.261	1.241	1.226	1.213	1.203	1.195	1.187	1.181	1.176	1.173	1.174	1.177	1.179	1.185	1.186	1.186	1.187	1.189
10.75	1.312		1.285		1.265	1.246	1.231	1.217	1.207	1.198	1.191	1.185	1.179	1.176	1.176	1.179	1.182	1.187	1.187	1.188	1.189	1.190
10.80	1.318		1.291		1.270	1.251	1.236	1.222	1.211	1.202	1.195	1.189	1.182	1.179	1.179	1.182	1.185	1.189	1.189	1.190	1.191	1.192
10.85	1.323		1.296		1.275	1.255	1.240	1.226	1.215	1.206	1.198	1.193	1.186	1.182	1.182	1.185	1.187	1.191	1.191	1.192	1.192	1.193
10.90	1.329		1.301		1.280	1.260	1.245	1.230	1.219	1.210	1.202	1.197	1.190	1.186	1.186	1.188	1.190	1.193	1.193	1.194	1.194	1.195
10.95	1.334		1.306		1.284	1.265	1.249	1.235	1.223	1.214	1.206	1.200	1.193	1.189	1.189	1.191	1.192	1.195	1.195	1.196	1.196	1.197
11.00	1.339		1.311		1.289	1.270	1.254	1.240	1.228	1.218	1.210	1.204	1.196	1.193	1.192	1.194	1.195	1.197	1.197	1.198	1.198	1.199
11.05	1.344		1.316		1.294	1.275	1.258	1.244	1.232	1.221	1.213	1.208	1.199	1.196	1.194	1.196	1.197	1.199	1.199	1.199	1.199	1.200
11.10	1.350		1.321		1.299	1.280	1.263	1.249	1.236	1.225	1.217	1.211	1.202	1.199	1.196	1.199	1.200	1.201	1.201	1.201	1.201	1.202
11.15	1.355		1.326		1.303	1.284	1.268	1.253	1.240	1.229	1.221	1.214	1.206	1.202	1.199	1.201	1.202	1.203	1.203	1.203	1.203	1.203
11.20	1.360		1.331		1.308	1.289	1.273	1.257	1.245	1.233	1.225	1.218	1.210	1.205	1.202	1.204	1.204	1.205	1.205	1.205	1.205	1.205
11.25	1.365		1.336		1.313	1.294	1.277	1.261	1.249	1.237	1.228	1.221	1.213	1.208	1.205	1.206	1.206	1.207	1.207	1.207	1.207	1.207
11.30	1.370		1.341		1.318	1.299	1.282	1.265	1.253	1.241	1.232	1.225	1.217	1.211	1.208	1.209	1.209	1.209	1.209	1.209	1.209	1.209
11.35	1.375		1.346		1.323	1.303	1.286	1.269	1.257	1.245	1.236	1.228	1.220	1.214	1.211	1.211	1.211	1.211	1.211	1.211	1.210	1.210
11.40	1.381		1.351		1.328	1.308	1.291	1.273	1.261	1.249	1.240	1.232	1.224	1.217	1.214	1.214	1.214	1.214	1.214	1.213	1.212	1.212
11.45	1.386		1.356		1.333	1.313	1.295	1.277	1.265	1.253	1.243	1.236	1.227	1.220	1.217	1.216	1.216	1.216	1.216	1.215	1.214	1.213
11.50	1.391		1.362		1.338	1.318	1.300	1.282	1.269	1.257	1.247	1.240	1.231	1.224	1.220	1.219	1.219	1.219	1.218	1.217	1.216	1.215
11.55	1.396		1.367		1.343	1.322	1.304	1.286	1.273	1.260	1.251	1.244	1.234	1.227	1.223	1.222	1.221	1.221	1.220	1.219	1.218	1.217
11.60	1.402		1.372		1.348	1.327	1.309	1.291	1.277	1.264	1.255	1.248	1.238	1.230	1.226	1.225	1.224	1.223	1.222	1.221	1.220	1.219
11.65	1.407		1.377		1.353	1.331	1.313	1.295	1.281	1.268	1.258	1.251	1.241	1.233	1.229	1.227	1.226	1.225	1.223	1.222	1.221	1.220
11.70	1.412		1.382		1.358	1.336	1.317	1.300	1.285	1.272	1.262	1.255	1.245	1.236	1.232	1.230	1.229	1.227	1.225	1.224	1.223	1.222

S: Smooth data.

0435

Standing and Katz Natural Gas Z Factor Data

PrVTr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
11.75	1.417		1.387		1.363	1.340	1.321	1.305	1.288	1.276	1.265	1.259	1.248	1.239	1.235	1.233	1.232	1.229	1.227	1.226	1.225	1.224
11.80	1.423		1.392		1.368	1.345	1.326	1.309	1.294	1.280	1.269	1.262	1.252	1.243	1.238	1.236	1.234	1.231	1.229	1.228	1.227	1.226
11.85	1.428		1.397		1.373	1.350	1.330	1.313	1.298	1.284	1.273	1.266	1.255	1.246	1.240	1.238	1.236	1.233	1.231	1.229	1.228	1.227
11.90	1.433		1.402		1.378	1.355	1.335	1.318	1.302	1.288	1.277	1.270	1.259	1.250	1.243	1.241	1.239	1.236	1.233	1.231	1.230	1.229
11.95	1.438		1.407		1.383	1.359	1.339	1.322	1.306	1.292	1.280	1.274	1.262	1.253	1.246	1.244	1.241	1.238	1.235	1.233	1.231	1.230
12.00	1.444		1.413		1.388	1.364	1.344	1.327	1.310	1.296	1.284	1.278	1.266	1.256	1.249	1.247	1.244	1.240	1.237	1.235	1.233	1.232
12.05	1.449		1.418		1.393	1.368	1.348	1.331	1.314	1.299	1.288	1.281	1.269	1.259	1.252	1.249	1.246	1.242	1.238	1.236	1.235	1.234
12.10	1.454		1.423		1.398	1.373	1.353	1.335	1.318	1.303	1.292	1.285	1.272	1.262	1.255	1.251	1.249	1.244	1.240	1.238	1.237	1.236
12.15	1.459		1.428		1.403	1.378	1.357	1.339	1.322	1.307	1.296	1.288	1.275	1.265	1.258	1.254	1.251	1.246	1.242	1.240	1.238	1.237
12.20	1.465		1.433		1.408	1.383	1.362	1.344	1.326	1.311	1.300	1.292	1.279	1.268	1.261	1.257	1.254	1.248	1.244	1.242	1.240	1.239
12.25	1.470		1.438		1.413	1.388	1.366	1.348	1.330	1.315	1.304	1.295	1.283	1.271	1.264	1.259	1.256	1.250	1.246	1.244	1.242	1.241
12.30	1.475		1.443		1.418	1.392	1.371	1.353	1.334	1.319	1.307	1.299	1.287	1.274	1.267	1.262	1.259	1.252	1.248	1.246	1.244	1.243
12.35	1.480		1.448		1.423	1.397	1.375	1.357	1.338	1.323	1.311	1.303	1.290	1.277	1.270	1.265	1.261	1.254	1.250	1.247	1.246	1.245
12.40	1.486		1.453		1.428	1.402	1.380	1.362	1.342	1.327	1.315	1.307	1.293	1.281	1.273	1.268	1.264	1.256	1.252	1.249	1.248	1.247
12.45	1.491		1.458		1.433	1.407	1.384	1.366	1.346	1.330	1.319	1.310	1.296	1.284	1.276	1.270	1.266	1.258	1.254	1.250	1.249	1.248
12.50	1.496		1.464		1.438	1.411	1.389	1.370	1.350	1.334	1.323	1.313	1.300	1.288	1.279	1.273	1.269	1.261	1.256	1.252	1.251	1.250
12.55	1.501		1.469		1.443	1.416	1.394	1.374	1.354	1.338	1.326	1.317	1.303	1.290	1.282	1.275	1.271	1.263	1.258	1.254	1.252	1.251
12.60	1.507		1.474		1.448	1.420	1.399	1.379	1.358	1.342	1.330	1.321	1.307	1.293	1.285	1.278	1.273	1.265	1.260	1.256	1.254	1.253
12.65	1.512		1.479		1.453	1.425	1.404	1.383	1.362	1.346	1.334	1.325	1.310	1.297	1.288	1.280	1.276	1.267	1.261	1.257	1.256	1.255
12.70	1.517		1.484		1.458	1.430	1.409	1.388	1.367	1.350	1.337	1.329	1.314	1.300	1.291	1.283	1.279	1.269	1.263	1.259	1.258	1.257
12.75	1.522		1.489		1.463	1.435	1.413	1.392	1.371	1.354	1.341	1.332	1.317	1.303	1.294	1.286	1.281	1.271	1.265	1.261	1.259	1.258
12.80	1.527		1.495		1.468	1.439	1.418	1.397	1.375	1.358	1.345	1.336	1.321	1.306	1.297	1.289	1.283	1.273	1.267	1.263	1.261	1.260
12.85	1.532		1.500		1.473	1.444	1.422	1.401	1.379	1.362	1.348	1.339	1.324	1.309	1.299	1.291	1.286	1.275	1.269	1.265	1.263	1.261
12.90	1.537		1.505		1.477	1.449	1.427	1.405	1.383	1.365	1.352	1.342	1.328	1.312	1.302	1.294	1.289	1.277	1.271	1.267	1.265	1.263
12.95	1.542		1.510		1.482	1.454	1.431	1.409	1.387	1.369	1.356	1.346	1.331	1.315	1.305	1.297	1.291	1.279	1.272	1.268	1.266	1.265
13.00	1.548		1.515		1.487	1.458	1.436	1.414	1.391	1.374	1.360	1.350	1.335	1.319	1.308	1.300	1.293	1.281	1.274	1.270	1.268	1.267
13.05	1.553		1.520		1.492	1.463	1.440	1.418	1.395	1.378	1.364	1.354	1.338	1.321	1.310	1.302	1.295	1.283	1.276	1.271	1.269	1.268
13.10	1.558		1.525		1.497	1.468	1.445	1.422	1.399	1.381	1.367	1.358	1.341	1.324	1.313	1.304	1.298	1.286	1.278	1.273	1.271	1.270
13.15	1.563		1.530		1.502	1.473	1.449	1.426	1.403	1.385	1.371	1.361	1.345	1.327	1.316	1.307	1.300	1.288	1.280	1.275	1.273	1.271
13.20	1.568		1.535		1.507	1.477	1.453	1.431	1.408	1.389	1.374	1.364	1.349	1.331	1.320	1.310	1.303	1.290	1.282	1.277	1.275	1.273
13.25	1.573		1.540		1.512	1.482	1.458	1.435	1.412	1.393	1.378	1.368	1.352	1.334	1.322	1.313	1.305	1.292	1.284	1.278	1.276	1.275
13.30	1.578		1.546		1.517	1.486	1.463	1.440	1.416	1.398	1.382	1.372	1.355	1.337	1.325	1.316	1.308	1.294	1.286	1.280	1.278	1.277
13.35	1.583		1.551		1.521	1.491	1.467	1.444	1.420	1.401	1.385	1.375	1.358	1.340	1.328	1.318	1.310	1.296	1.287	1.281	1.279	1.278

S: Smooth data.

Standing and Katz Natural Gas Z Factor Data

PrTr	1.05	1.05(S)	1.1	1.1(S)	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	2.6	2.8	3
13.40	1.589		1.556		1.526	1.495	1.472	1.449	1.424	1.404	1.389	1.379	1.362	1.343	1.331	1.321	1.313	1.298	1.289	1.283	1.281	1.280
13.45	1.594		1.561		1.531	1.500	1.476	1.453	1.428	1.408	1.393	1.382	1.365	1.346	1.334	1.323	1.315	1.300	1.291	1.285	1.283	1.281
13.50	1.599		1.566		1.536	1.504	1.481	1.457	1.432	1.412	1.397	1.386	1.369	1.349	1.337	1.326	1.317	1.302	1.293	1.287	1.285	1.283
13.55	1.604		1.571		1.541	1.509	1.485	1.461	1.436	1.416	1.400	1.389	1.372	1.352	1.340	1.328	1.319	1.304	1.295	1.289	1.287	1.285
13.60	1.609		1.577		1.546	1.514	1.490	1.466	1.440	1.420	1.404	1.393	1.376	1.355	1.343	1.331	1.322	1.306	1.297	1.291	1.289	1.287
13.65	1.614		1.582		1.551	1.518	1.494	1.470	1.444	1.424	1.408	1.396	1.379	1.358	1.346	1.334	1.324	1.308	1.298	1.292	1.290	1.288
13.70	1.619		1.587		1.556	1.523	1.499	1.475	1.448	1.428	1.412	1.400	1.382	1.361	1.349	1.337	1.327	1.310	1.300	1.294	1.292	1.290
13.75	1.624		1.592		1.561	1.527	1.503	1.479	1.452	1.432	1.416	1.404	1.385	1.364	1.351	1.339	1.329	1.312	1.301	1.296	1.293	1.291
13.80	1.630		1.597		1.566	1.532	1.508	1.483	1.456	1.436	1.419	1.408	1.389	1.368	1.354	1.342	1.332	1.314	1.303	1.298	1.295	1.293
13.85	1.635		1.602		1.571	1.537	1.513	1.487	1.460	1.439	1.423	1.411	1.393	1.371	1.357	1.345	1.334	1.316	1.305	1.300	1.297	1.295
13.90	1.640		1.608		1.575	1.542	1.517	1.492	1.465	1.443	1.426	1.414	1.397	1.374	1.360	1.348	1.337	1.318	1.307	1.301	1.299	1.297
13.95	1.645		1.613		1.580	1.546	1.522	1.496	1.469	1.447	1.430	1.417	1.399	1.377	1.363	1.350	1.339	1.320	1.309	1.302	1.300	1.298
14.00	1.650		1.618		1.585	1.550	1.526	1.500	1.473	1.451	1.434	1.421	1.402	1.380	1.366	1.353	1.341	1.322	1.311	1.304	1.302	1.300
14.05	1.655		1.623		1.590	1.555	1.531	1.504	1.478	1.455	1.437	1.425	1.405	1.383	1.369	1.356	1.343	1.324	1.313	1.306	1.303	1.301
14.10	1.661		1.628		1.595	1.560	1.535	1.509	1.482	1.459	1.441	1.429	1.409	1.386	1.372	1.358	1.346	1.326	1.315	1.308	1.305	1.303
14.15	1.666		1.633		1.600	1.565	1.539	1.513	1.486	1.463	1.445	1.432	1.412	1.389	1.375	1.360	1.348	1.328	1.317	1.309	1.307	1.305
14.20	1.671		1.639		1.605	1.570	1.544	1.518	1.490	1.467	1.449	1.436	1.416	1.392	1.378	1.363	1.351	1.330	1.319	1.311	1.309	1.307
14.25	1.676		1.644		1.609	1.575	1.548	1.522	1.494	1.471	1.452	1.439	1.419	1.395	1.381	1.366	1.353	1.332	1.320	1.313	1.310	1.308
14.30	1.681		1.649		1.614	1.580	1.553	1.527	1.498	1.475	1.456	1.443	1.423	1.398	1.384	1.369	1.356	1.334	1.322	1.315	1.312	1.310
14.35	1.686		1.654		1.619	1.584	1.557	1.531	1.502	1.478	1.460	1.446	1.426	1.401	1.387	1.371	1.358	1.336	1.323	1.316	1.313	1.311
14.40	1.692		1.659		1.624	1.589	1.561	1.536	1.506	1.482	1.464	1.450	1.430	1.404	1.390	1.374	1.360	1.338	1.325	1.318	1.315	1.313
14.45	1.697		1.664		1.629	1.594	1.565	1.540	1.510	1.486	1.468	1.454	1.433	1.407	1.393	1.376	1.362	1.340	1.327	1.320	1.317	1.315
14.50	1.702		1.669		1.634	1.598	1.570	1.544	1.515	1.490	1.472	1.458	1.437	1.411	1.396	1.379	1.365	1.342	1.329	1.322	1.319	1.317
14.55	1.707		1.674		1.639	1.603	1.575	1.548	1.520	1.494	1.476	1.461	1.440	1.414	1.398	1.382	1.367	1.344	1.331	1.324	1.321	1.318
14.60	1.712		1.679		1.643	1.608	1.580	1.552	1.524	1.498	1.480	1.465	1.443	1.417	1.400	1.385	1.370	1.346	1.333	1.326	1.323	1.320
14.65	1.717		1.684		1.648	1.613	1.584	1.556	1.528	1.502	1.484	1.468	1.446	1.420	1.403	1.387	1.372	1.348	1.334	1.327	1.324	1.321
14.70	1.722		1.690		1.653	1.617	1.589	1.561	1.532	1.505	1.488	1.472	1.450	1.423	1.407	1.390	1.375	1.350	1.336	1.329	1.326	1.323
14.75	1.727		1.695		1.658	1.622	1.594	1.565	1.536	1.509	1.491	1.475	1.453	1.426	1.410	1.392	1.377	1.352	1.338	1.330	1.327	1.325
14.80	1.733		1.700		1.663	1.627	1.598	1.570	1.540	1.513	1.495	1.479	1.457	1.429	1.413	1.395	1.380	1.354	1.340	1.332	1.329	1.327
14.85	1.738		1.705		1.668	1.632	1.603	1.574	1.544	1.517	1.498	1.483	1.460	1.432	1.416	1.398	1.382	1.356	1.341	1.334	1.331	1.328
14.90	1.743		1.710		1.673	1.636	1.607	1.579	1.549	1.521	1.502	1.487	1.463	1.436	1.419	1.401	1.385	1.358	1.343	1.336	1.333	1.330
14.95	1.748		1.715		1.678	1.641	1.612	1.583	1.553	1.525	1.505	1.490	1.466	1.439	1.421	1.403	1.387	1.360	1.345	1.338	1.334	1.331
15.00	1.753		1.720		1.682	1.645	1.616	1.588	1.558	1.529	1.508	1.493	1.470	1.442	1.424	1.406	1.390	1.362	1.347	1.340	1.336	1.333

S: Smooth data.

5170

APPENDEX-B

The following statistics were used in this study. The parameters calculated were average percent relative error. average percent absolute relative error, minimum/maximum absolute percent relative error. the root mean square error, standard deviation, skewness. kurtosis, the coefficient of determination and the coefficient of correlation.

Average Percent Relative Error:

The average percent relative error is defined as:

$$E_r = (\frac{1}{n})\sum_{i=1}^{n} E_i$$

E_i is the relative deviation in percent of an estimated value from a measured value and is defined by:

$$E_i = (\frac{x_{exp} - x_{est}}{x_{exp}})*100, \quad i = 1,2,....n$$

Where x_{exp} and x_{est} represent the experimental and estimated values respectively. E_r is an indication of the relative deviation in percent from the experimental values. The lower the value of E_r, the more equally distributed are the errors between positive and negative values.

Average Absolute Percent Relative Error:

The average absolute percent relative error is defined as:

$$E_a = (\frac{1}{n})\sum_{i=1}^{n} |E_i|$$

E_a indicates the relative absolute deviation in percent from the experimental values. The lower the error, the better the correlation.

Minimum/Maximum Absolute Percent Relative Error:

After calculating the absolute percent relative error for each data point, $|E_i|$, $I= 1,2,... n_d$, both the minimum and maximum values are determined to estimate the range of error for each correlation:

$$E_{min} = \underset{i=1}{\overset{n}{Min}}|E_i|$$

$$E_{max} = \underset{i=1}{\overset{n}{Max}}|E_i|$$

The accuracy of a correlation is determined also by the maximum absolute percent relative error. The lower the value of the maximum absolute percent relative error. the higher is the accuracy of the correlation.

The Root Mean Square Error:

It is a measure of the closeness of correlation prediction to the measured values and is defined as follows:

$$E_{RMS} = \sqrt{\sum_{i=1}^{n} (E_i)^2 / n}$$

The lower the value of E_{RMS}, the better the correlation. As the value goes higher, a less favorable fit is obtained.

Standard Deviation:

The standard deviation of the data 's' is a reflection of the dispersion of the data around the mean. It is expressed as the square root of the variance s^2:

$$s^2 = (\frac{1}{n-1})\sum_{i=1}^{n} (x_i - \bar{x})^2$$

Where $\bar{x}$ is the mean defined as:

$$\bar{x} = (\frac{1}{n})\sum_{i=1}^{n} x_i$$

The lower the value of the standard deviation, the smaller the degree of dispersion of the data.

Skewness:

Skewness is a measure of asymmetry of a distribution. The distribution is said to be skewed to the right or to have positive skewness when the frequency curve has a longer tail to the right. Otherwise, when the longer tail of the frequency distribution is to the left the data is then skewed to the left or often called to have negative skewness.

$$Skewness = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s}\right)^3$$

Kurtosis:

Kurtosis is a measure of the degree of peakedness of a distribution. For a high peak distribution kurtosis will have a positive value while for flat one it will have a negative value. If the distribution is normal, kurtosis will have a zero value. Kurtosis is defined by the following relation:

$$Kurtosis = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s}\right)^4\right] - 3$$

The Coefficient of Variation:

The coefficient of variation '$C.V.$' is used to describe the amount of variation in the population as a proportion of the mean. It is defined as:

$$C.V. = \frac{E_{RMS}}{\bar{x}} * 100$$

The coefficient of variation is dimensionless and thus it is often calculated as an evaluation parameter. The lower the value of $C.V.$, the better the correlation.

The Correlation Coefficient:

The correlation coefficient 'R' represents the degree of success in reducing the standard deviation by regression analysis. On the other hand, the coefficient of determination is simply the square of the correlation coefficient and defined by:

$$R^2 = 1 - \sum_{i=1}^{n} (x_{exp} - x_{est})_i^2 / \sum_{i=1}^{n} (x_{exp} - \bar{x})_i^2$$

Where $\bar{x}$ is the mean defined as:

$$\bar{x} = (\frac{1}{n}) \sum_{i=1}^{n} (x_{exp})_i$$

The correlation coefficient lies between 0 and 1. A value of 1 indicates a perfect correlation whereas a value of 0 implies no correlation at all among the given independent variables. The larger the value of r, the greater is the reduction in the sum of squares of errors, and the stronger is the relationship between the independent variable and the dependent ones.

NOMENCLATURE

Symbol	Description
P_r	Pseudoreduced pressure
T_r	Pseudoreduced temperature
y	The product of a van der Waals covolume and density
b	Van der Waals covolume
Z	Gas compressibility factor
R	Universal gas constant
ρ_r	Gas reduced density
ρ	Gas density, lb moles/ft^3
P_c	Critical pressure, psia
T_c	Critical Temperature, $^\circ$ R
Z_c	Gas compressibility factor at the critical
P_{pc}	Pseudocritical pressure, psia
T_{pc}	Pseudocritical temperature, $^\circ$ R
s	Standard deviation
E_r	Average percent relative error
E_a	Average absolute percent relative error
E_{min}	Minimum absolute percent relative error
E_{max}	Maximum absolute percent relative error
E_{RMS}	Root mean square error
$C.V.$	Coefficient of variation
R^2	Coefficient of determination
i	Index
j	Index

REFERENCES

1. Standing, M. B. and Katz, D. L., "Density of Natural Gases", Trans. AIME, Vol. 146, 1942, pp. 140-149.

2. Dranchuk, P. M. and Abou-Kassem, J. H., "Calculation of Z factors For Natural Gases Using Equations of State", Journal of Canadian Petroleum Technology, July-September, 1975, pp. 34-36.

3. Takacs G., "Comparing methods for calculating Z-factor", Oil & Gas Journal, May 15, 1989, pp. 43-46.

4. Katz, D.L., and Rzasa M. J., "Bibliography for Physical Behavior of Hydrocarbons Under Pressure," J. W. Edwards, Publisher, Inc., Ann Arbor, Mich., 1946.

5. Brown, G. G., and Holcomb D. E., "The Compressibility of Gases and Vapor - Liquid Phase Equilibria in Hydrocarbon Systems", Petrol. Engineering, II (5): 23, 1940.

6. Brown, G. G., Katz, D. L., Oberfell, G. G., and Alden, R. C., " Natural Gasoline and The Volatile Hydrocarbons", NGAA, Tulsa, pp. 23-56, 1948.

7. Dranchuk, P. M. and Quon, D., "A general solution of the equations describing steady state turbulent compressible flow in circular conduits", Journal of Canadian Petroleum Technology, Summer 1965, pp. 60-65.

8. Papay, J., "A termelestechnologiai parameterek valtozasa a gaztelepek muvelese soran", OGIL Musz. Tud Kozl., Budapest, 1968, pp. 267-273.

9. Gopal, V. N., " Gas Z-factor equations developed for computer". The Oil and Gas Journal, August 8, 1977, pp. 58-60.

10. Burnett, R. R., "Calculator gives compressibility factors", The Oil and Gas Journal, June 11, 1979, pp. 70-74.

11. Papp Istvan, "Uj modszer foldgazok elteresi tenyezojenek szamitasara," Koolaj es Foldgaz 12. (112.) evfolyam 11. szam, November 1979, pp. 345-47.

12. Hankinson, R. W., Thomas, L. K. and Phillips, K. A., "Predict Natural Gas properties", Hydrocarbon Processing, April 1969, pp. 106-108.

13. Poettman, F. H., Carpenter, P. G., Drilling and Production Practice, American Petroleum Institute, New York (1953).

14. Yarborough, L. and Hall, K. R., "A new equation of state for Z-factor calculations", The Oil and Gas Journal, June 18, 1973, pp. 82-92.

15. Yarborough, L. and Hall, K. R., "How to solve equation of state for Z-factors", The Oil and Gas Journal, February 18, 1974, pp. 86-88.

16. Wichert, E. and Aziz K., "Compressibility Factor of Sour Natural Gases", Can. J of Che., Vol. 49, pp. 267-273 (1971).

17. Starling, K. E., Fluid Thermodynamic Properties For Light Petroleum Systems, Gulf Publishing Company, Houston, pp. 1-3, 1973.

18. Dranchuk. P. M.. Purvis, R. A. and Robinson, D. B., "Computer calculations of Natural Gas compressibility factors using the Standing and Katz correlation", Inst. of Petroleum Technical Series, No. IP 74-008. 1974.

19. Sage, Bruce H.. and William N. Lacey. Volumetric and Phase Behaviour of Hydrocarbons, Gulf Publishing Company, Houston, Tex., 1949.

20. Davis, P. M., Bertuzzi, A. F., Gore, T. L., and Kurata, F., The Phase and Volumetric Behaviour of Natural Gases at Low Temperatures and High Pressures, Trans. AIME, Vol. 201, pp. 245, 1954.

21. Katz, D. L., *et al.*, Handbook of Natural Gas Engineering, McGraw-Hill, pp. 710-717, 1959.

VITA

PERSONAL INFORAMTION:

Name:	Mohammed Najim Al-Khamis
Date of Birth:	May 09, 1960
Marital Status:	Married, Two kids
Permanent Address:	Saudi ARAMCO, Ras Tanura 31311
	P. O. Box 1997

EDUCATION:

1. **M.Sc. in Petroleum Engineering,** (March 1995), King Fahd University of Petroleum & Minerals. Thesis Title: " Evaluation of Correlations for Natural Gas Compressibility Factors".

2. **B.Sc. in Petroleum Engineering,** (July 1988), King Fahd University of Petroleum & Minerals. Rate of Appreciation: Distinction with Honors.

WORK EXPERIENCE:

Worked in various divisions: Production Engineering, Reservoir Engineering and Drilling Operations of Saudi Aramco company since 1988. Currently, working with Northern Area Production Engineering.

MEMBERSHIP OF PROFESSIONAL SOCIETIES:

SPE, American Society of Petroleum Engineers.